SVOLGIMENTO PROVA SCRITTA di ANALISI 1 del 14/7/2017

A1

COMPITO A

1) f è definita $\forall x \in \mathbb{R}$.

CONTINUITÀ:

$$\frac{-\sin(2x)+2x}{4x^3} = \frac{-\left(2x - \frac{(2x)^3}{6} + o(x^3)\right) + 2x}{4x^3}$$

$$= \frac{\frac{4}{3}x^3 + o(x^3)}{4x^3} \xrightarrow[x \to 0^-]{} \frac{1}{3}$$

$$\frac{e^{2x^2}-1-2x^2}{6x^b} = \frac{\left(2x^2 + \frac{(2x^2)^2}{2} + o(x^4)\right) - 2x^2}{6x^b}$$

$$= \frac{2x^4 + o(x^4)}{6x^b} \xrightarrow[x \to 0^+]{} \frac{1}{3} \iff \underline{\underline{b=4}}$$

$$\lim_{x \to 0} f(x) = \frac{1}{3} = f(0) \iff a = \frac{1}{3}.$$

$$\Rightarrow f \in C^0(\mathbb{R})$$

DERIVABILITÀ:

A_2

$$f'(x) = \begin{cases} \dfrac{1}{4}\left[\dfrac{(-2\cos(2x)+2)x^{\cancel{3}} - (-\sin(2x)+2x)3\cancel{x^2}}{\cancel{3}x^{\cancel{6}\,4}}\right] & x<0 \\[2ex] \dfrac{1}{6}\left[\dfrac{(4x e^{2x^2}-4x)\,x^{\cancel{4}} - (e^{2x^2}-1-2x^2)4\cancel{x^3}}{x^{\cancel{8}\,5}}\right] & x>0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{4}\left[\dfrac{2x(1-\cos(2x)) + 3(\sin(2x)-2x)}{x^4}\right] & x<0 \\[2ex] \dfrac{2}{3}\left[\dfrac{(e^{2x^2}-1)\cancel{(x^2-1)}\,x^2 - (e^{2x^2}-1-2x^2)}{x^5}\right] & x>0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{4}\left[\dfrac{2x\left[2x^2 - \dfrac{(2x)^4}{4!} + o(x^4)\right] + 3\left[-\dfrac{(2x)^3}{3!} + \dfrac{(2x)^5}{5!} + o(x^5)\right]}{x^4}\right] & x<0 \\[2ex] \dfrac{2}{3}\left[\dfrac{\left(2x^2 + \dfrac{(2x^2)^2}{2} + o(x^4)\right)x^2 - \left(\dfrac{(2x^2)^2}{2} + \dfrac{(2x^2)^3}{3!} + o(x^6)\right)}{x^5}\right] \end{cases}$$

$$= \begin{cases} \dfrac{1}{4x^4}\left[\cancel{4x^3} - \dfrac{4}{3}x^5 - \cancel{4x^3} + \dfrac{4}{5}x^5 + o(x^5)\right] & x<0 \\ \dfrac{2}{3x^5}\left[\cancel{2x^4} + 2x^6 - \cancel{2x^4} - \dfrac{4}{3}x^6 + o(x^6)\right] & x>0 \end{cases}$$ (A3)

$$= \begin{cases} -\dfrac{2}{15}x + o(x) & x<0 \\ \dfrac{4}{9}x + o(x) & x>0 \end{cases}$$

$$\lim_{x\to 0^\pm} f'(x) = 0 \quad \Rightarrow \quad f'(0)=0$$

f DERIVABILE in $\mathbb{R}$

per $a=\dfrac{1}{3}$; $b=4$.

In alternativa sviluppando ulteriormente i polinomi di Mc Laurin:

$$f(x) = \begin{cases} \dfrac{\dfrac{4}{3}x^3 - \dfrac{4}{15}x^5 + o(x^5)}{4x^3} = \dfrac{1}{3} - \dfrac{1}{15}x^2 + o(x^2) & x<0 \\ \dfrac{2x^4 + \dfrac{4}{3}x^6 + o(x^6)}{6x^4} = \dfrac{1}{3} + \dfrac{2}{9}x^2 + o(x^2) & x>0 \end{cases}$$

A4

$$\Rightarrow f'(x) \sim \begin{cases} -\dfrac{2}{15}x & x<0 \\ \dfrac{4}{9}x & x>0 \end{cases}$$

$$\Rightarrow \lim_{x \to 0^\pm} f'(x) = 0$$

2) f pari

$$\Rightarrow \int_{-1}^{1} f(x)\,dx = 2\int_0^1 f(x)\,dx$$

$$= 2\int_0^1 (2x^4+3x)\,dx \qquad (\text{in quanto } x \geq 0)$$

$$= 2\left[\frac{2}{5}x^5 + \frac{3}{2}x^2\right]_0^1 = 2\left(\frac{2}{5}+\frac{3}{2}\right) = \frac{19}{5}.$$

3)

$$z = \left[\frac{2^5\left(\cos\left(\frac{10}{3}\pi\right) + i\sin\left(\frac{10}{3}\pi\right)\right)}{\left(\cos(4\pi)+i\sin(4\pi)\right)e^{i\pi}}\right]$$

$$= 2^5\left[\frac{\cos\left(-\frac{2}{3}\pi\right)+i\sin\left(-\frac{2}{3}\pi\right)}{-1}\right]$$

$$= 2^5 \left[-\cos\left(-\frac{2}{3}\pi\right) - i \operatorname{sen}\left(-\frac{2}{3}\pi\right) \right]$$

$$= 2^5 \left[\cos\left(\frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{3}\right) \right]$$

$$= 2^5 \left[\frac{1}{2} + i \frac{\sqrt{3}}{2} \right] = 2^4 \left(1 + i\sqrt{3}\right)$$

4) OMOGENEA ASSOCIATA:

$$y'' - y' = 0 \quad \Rightarrow \alpha^2 - \alpha = 0$$

$$\Rightarrow \alpha_1 = 0 \; ; \; \alpha_2 = 1$$

$$y_0(x) = C_1 + C_2 e^x$$

NON OMOGENEA: poichè $\alpha = 0$ è radice del polinomio caratteristico, allora

$$y_p(x) = x(Ax + B)$$

$$y_p'(x) = 2Ax + B$$

$$y_p''(x) = 2A$$

A 6

$$2A - 2Ax - B = 4 - 3x$$

$$\Rightarrow \begin{cases} A = \dfrac{3}{2} \\ B = 2A - 4 = -1 \end{cases}$$

$$\Rightarrow y(x) = C_1 + C_2 e^x + \frac{3}{2}x^2 - x$$

$$\lim_{x \to -\infty} y(x) = \lim_{x \to -\infty} \frac{3}{2}x^2 = +\infty$$

$$\forall C_1, C_2 \in \mathbb{R}$$

$$\lim_{x \to +\infty} y(x) = \begin{cases} \lim\limits_{x \to +\infty} C_2 e^x & \text{per } C_2 \neq 0 \\ \lim\limits_{x \to +\infty} \frac{3}{2}x^2 & \text{per } C_2 = 0 \end{cases}$$

$$\Rightarrow \lim_{x \to +\infty} y(x) = +\infty \qquad \forall C_1 \in \mathbb{R};\ \forall C_2 \geq 0.$$

5) La serie è definita per A 7

$$\left| \frac{1+\sqrt{3}n}{3+2n} \right| \leq 1$$ cioè

$$\frac{1+\sqrt{3}n}{3+2n} \leq 1 \iff 1+\sqrt{3}n \leq 3+2n$$

$$\iff \underbrace{(2-\sqrt{3})}_{>0} n \geq -2$$ verificata $\forall n \in \mathbb{N}$

Convergenza assoluta:

$$|a_n| = \left| \arccos\left(\frac{1+\sqrt{3}n}{3+2n}\right) \right|^n$$

Criterio della radice

$$\sqrt[n]{|a_n|} = \left| \arccos\left(\frac{1+\sqrt{3}n}{3+2n}\right) \right| \xrightarrow[n\to\infty]{} \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} < 1.$$

Ne segue che la serie converge assolutamente e quindi semplicemente.