SVOLGIMENTO PROVA SCRITTA di ANALISI MAT. 1 del 19/6/2015 A_1

COMPITO A

1) $z \neq 2+i$

$$(z-i+2)(z-i-2)=2iz+3$$

$$\left[(z-i)^2-4\right]=2iz+3$$

$$z^2-4iz-8=0$$

$$z_{1,2}=2i\pm\sqrt{-4+8}=\pm 2+2i$$

$$z_1=2\sqrt{2}\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right]=2\sqrt{2}\left[\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right]$$

$$=2\sqrt{2}\,e^{i\frac{\pi}{4}}$$

$$z_2=2\sqrt{2}\left[-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right]=2\sqrt{2}\left[\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)\right]$$

$$=2\sqrt{2}\,e^{i\frac{3}{4}\pi}$$

2) $$\sqrt{\frac{n^2+2n+3}{n^2+2}}-1=\sqrt{1+\frac{2n+1}{n^2+2}}-1$$

$$\sim\frac{2n+1}{2(n^2+2)}\sim\frac{1}{n}$$

$$\Rightarrow \sum a_n \approx \sum \frac{n^\alpha}{n}=\sum\frac{1}{n^{1-\alpha}} \quad \text{convergente}$$

se $1-\alpha>1 \iff \alpha<0$

3) In forma normale: A_i

$$y' + 2xy = \frac{e^{-x^2}}{\sqrt{1-x^2}}$$

l'equazione è definita per $x \in (-1,1)$.
Si può risolvere l'equazione (lineare del 1° ordine) con la formula risolutiva, oppure riscrivendola nella forma

$$\left(e^{x^2} y\right)' = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow e^{x^2} y = \int \frac{1}{\sqrt{1-x^2}}\, dx + C$$

$$\Rightarrow y = e^{-x^2}\left[\arcsin x + C\right]$$

$y(0) = 1 = C \Rightarrow$ la sol. è

$$y(x) = e^{-x^2}\left[\arcsin x + 1\right]$$

Poiché $a(x) = 2x \in C^\infty(\mathbb{R})$

$$f(x) = \frac{e^{-x^2}}{\sqrt{1-x^2}} \in C^\infty(-1,1)$$

$\Rightarrow \exists !$ sol. $y \in C^1(-1,1)$.
(sol. GLOBALE)

4) $\cancel{\log(1+x^2) - x}$ $f(x)$ è definita per $tg(x^4) \neq 0$, cioè per

$$\begin{cases} x^4 \neq \frac{\pi}{2} + k\pi & ; \ k \in \mathbb{N}_0 \\ x^4 \neq h\pi & ; \ h \in \mathbb{N}_0 \end{cases}$$

$$\Rightarrow \left\{ x \neq \pm\sqrt[4]{\frac{\pi}{2} + k\pi} \ ; \ k \in \mathbb{N}_0 \right\}$$

$$\cap \left\{ x \neq \pm\sqrt[4]{h\pi} \ ; \ h \in \mathbb{N}_0 \right\}$$

$$\frac{\log(1+x^2) - x \sin x}{tg(x^4)} = \frac{x^2 - \frac{x^4}{2} + o(x^4) - x\left[x - \frac{x^3}{6} + o(x^3)\right]}{x^4 + o(x^4)}$$

$$\sim \frac{\cancel{x^4}\left(\frac{1}{6} - \frac{1}{2}\right)}{\cancel{x^4}} = -\frac{1}{3} \xrightarrow[x \to 0]{} -\frac{1}{3}.$$

5) È sufficiente studiare la funzione

$$g(x) = e^x(x-2)$$

e poi considerarne il modulo.

$$D = \mathbb{R}$$

$\lim_{x \to +\infty} g(x) = +\infty$ (andamento superlineare $\Rightarrow$ NO AS. OBLIQUO)

$\lim_{x \to -\infty} g(x) = 0^-$ (AS. ORIZZ. $y = 0$) A4

$g(x) = 0 \iff x = 2$

$g(x) > 0 \iff x > 2$

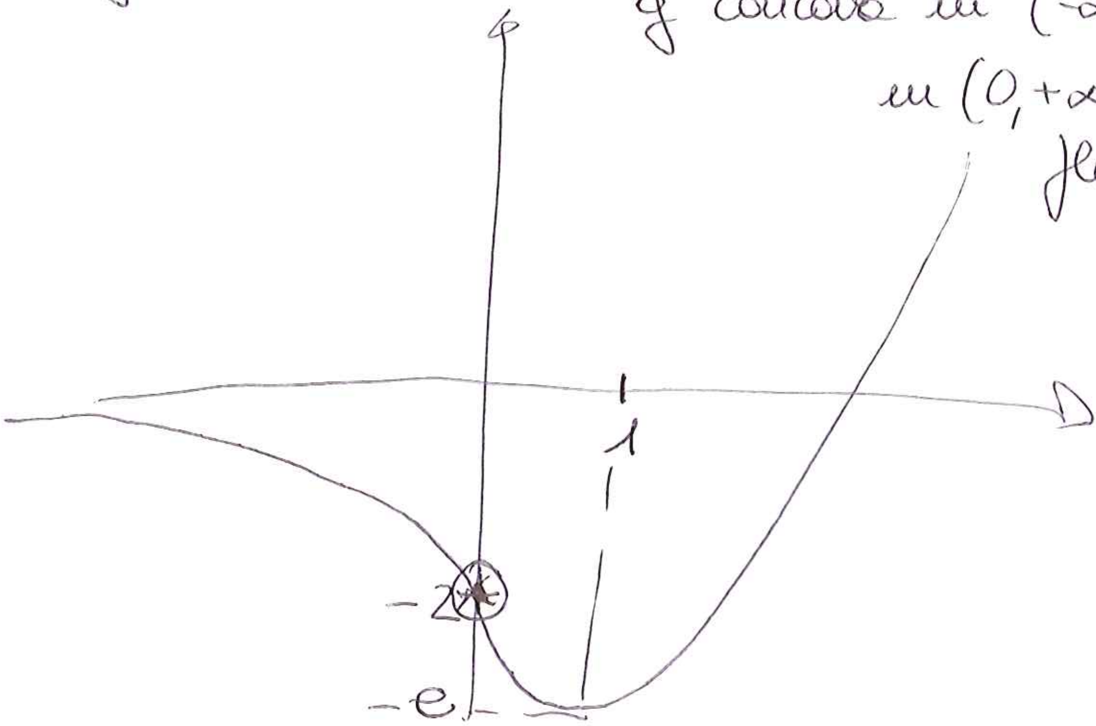
$g(0) = -2$

$g'(x) = e^x(x-1) > 0 \iff x > 1$

g decresce in $(-\infty, 1)$; cresce in $(1, +\infty)$. $x = 1$ punto di MIN $g(1) = -e$

$g''(x) = e^x x > 0 \iff x > 0$

g concava in $(-\infty, 0)$; convessa in $(0, +\infty)$; punto di flesso: $x = 0$ $g(0) = -2$



$\Rightarrow \quad f(x) = |g(x)|$

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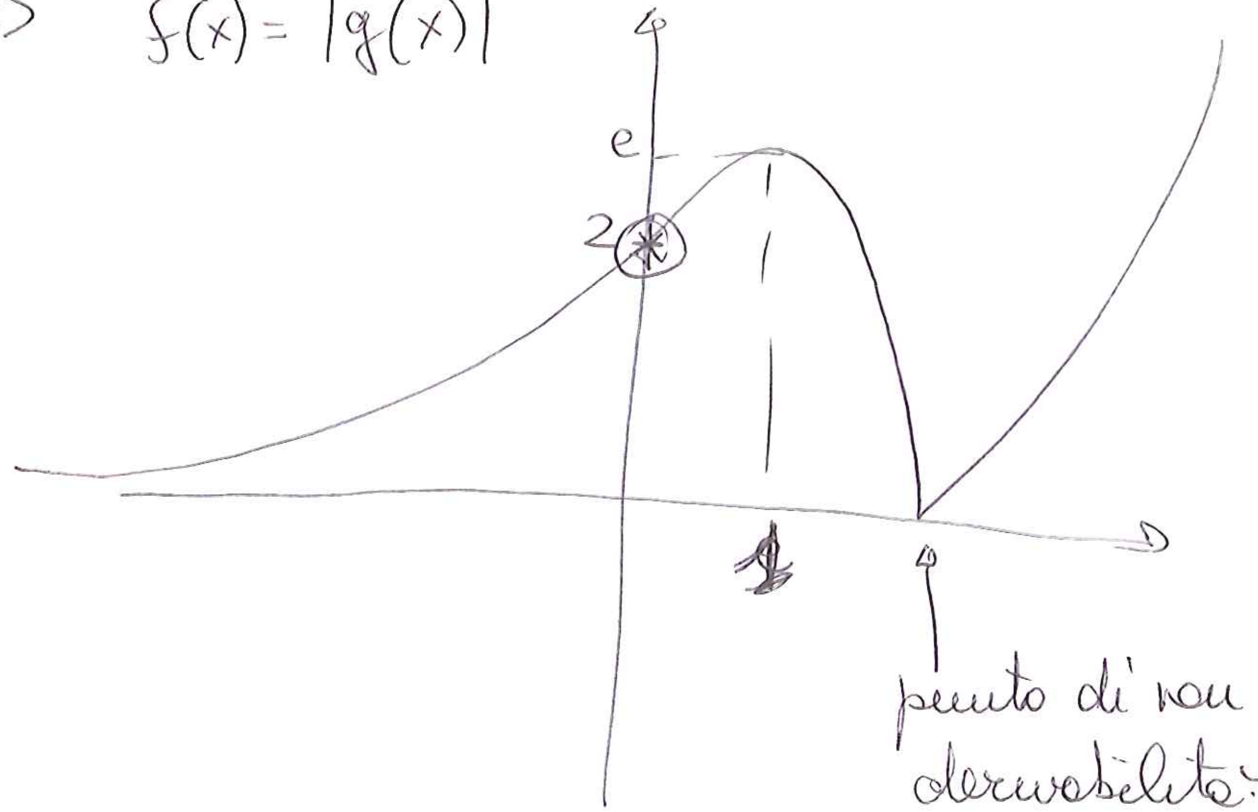

In $[0, +\infty)$:

in $x = 0$ MIN. REL. $f(0) = 2$

in $x = 1$ MAX. REL. $f(1) = e$

in $x = 2$ MIN. REL. e ASS. $f(2) = 0$

$\nexists$ MAX. ASS.: $\lim_{x \to +\infty} f(x) = +\infty$.