SVOLGIMENTI PROVA SCRITTA di ANALISI MAT. 1 del 17/3/2017

1) $\log^3\left(1+\frac{1}{n}\right) \sim \left(\frac{1}{n}\right)^3$

$\sin^2\left[\log^3\left(1+\frac{1}{n}\right)\right] \sim \sin^2\left[\left(\frac{1}{n}\right)^3\right] \sim \frac{1}{n^6}$

$\operatorname{arctg}\left[\sin^2\left(\log^3\left(1+\frac{1}{n}\right)\right)\right] \sim \operatorname{arctg}\left(\frac{1}{n^6}\right)$

$\sim \frac{1}{n^6}$

$\Rightarrow \sqrt{\operatorname{arctg}\left[\sin^2\left(\log^3\left(1+\frac{1}{n}\right)\right)\right]} \sim \frac{1}{n^3}$

$\Rightarrow$ la serie converge per confronto con la serie armonica $\sum \frac{1}{n^3}$.

2) $3e^{2x}-2e^x+5>0 \qquad e^x=t$

$3t^2-2t+5>0 \qquad t_{1,2}=1\pm\sqrt{1-15}$

sempre verificato.

f è definita su tutto $\mathbb{R}$.

$$f'(x) = -\frac{1}{3e^{2x}-2e^{x}+5}\left(6e^{2x}-2e^{x}\right)$$ ②

$$= \frac{2e^{x}(1-3e^{x})}{3e^{2x}-2e^{x}+5} \geq 0$$

$$\iff 1-3e^{x} \geq 0 \iff e^{x} \leq \frac{1}{3}$$

$$\iff x \leq -\log 3$$

f ~~decres~~ cresce in $[-\log 6, -\log 3)$,
decresce in $(-\log 3, 0]$.

$x = -\log 3$ punto di MAX. REL. e ASS.

$$f(-\log 3) = \log 6 - \log\left[\frac{\cancel{3}^{1}}{\cancel{9}_{3}} - \frac{2}{3} + 5\right]$$

$$= \log 6 - \log\left(\frac{14}{3}\right) = \log\left(\frac{9}{7}\right)$$

$x = -\log 6$ punto di MIN. REL.

$$f(-\log 6) = \log 6 - \log\left(\frac{\cancel{3}^{1}}{\cancel{36}_{12}} - \frac{\cancel{2}^{1}}{\cancel{6}_{3}} + 5\right)$$

$$= \log 6 - \log\left(\frac{1-4+60}{12}\right)$$

$$= \log\left(\frac{\cancel{6}^{2}\cdot 12}{\cancel{57}_{19}}\right) = \log\left(\frac{24}{19}\right)$$

$x=0$ punto di MIN. REL.

$f(0)=\log 6-\log(3-2+5)=0$

Poiché $\log\left(\frac{24}{19}\right)>0 \implies$

$x=0$ punto di MIN. ASS.

3) Per $x\to 0^+$: $\operatorname{arctg}x \sim x$

$$\implies f(x)\sim \frac{x}{x^{4/3}}=\frac{1}{x^{1/3}} \quad \text{integrabile in } (0,a]$$

Per $x\to+\infty$

$$0<\frac{\operatorname{arctg}x}{x^{4/3}}<\frac{\pi}{2x^{4/3}} \quad \text{integrabile in } [a,+\infty)$$

$\implies$ l'integrale converge.

4) $2\,\mathrm{Re}(i(x-iy))\cdot \mathrm{Im}(i(x+iy))$

$$-\cancel{\sqrt{\ }}\,x^2+y^2=0$$

$$2\,\mathrm{Re}(y+ix)\cdot \mathrm{Im}(-y+ix)-\cancel{\sqrt{\ }}\,x^2+y^2=0$$

$$2y\cdot x-\cancel{\sqrt{\ }}\,x^2+y^2=0$$

$$x^2+y^2-2xy=0$$

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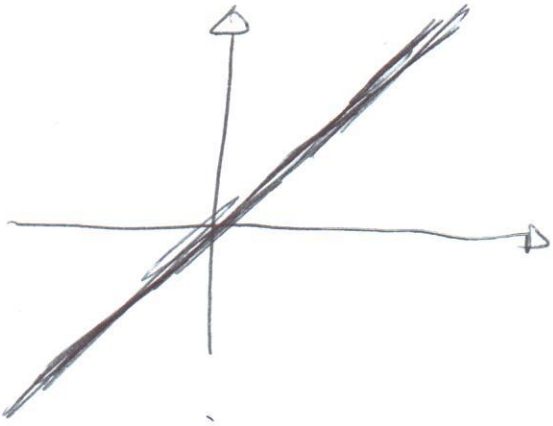
$$(y-x)^2=0$$

$$\Rightarrow \quad y=x$$



$$2\,\mathrm{Re}(i\bar{z})\cdot \mathrm{Im}(iz) \leq |z^2|$$

$$\Rightarrow 2xy \leq x^2+y^2$$

$$\Rightarrow (y-x)^2 \geq 0 \quad \Rightarrow \quad z \in \mathbb{C}.$$

5) ~~OM~~O. ASSOCIATA:

$$4\alpha^2-4\alpha+1=0$$

$$(2\alpha-1)^2=0 \quad \Rightarrow \alpha=\frac{1}{2} \quad m_a=2$$

$$\Rightarrow y_0(x)=C_1 e^{\frac{1}{2}x}+C_2 x e^{\frac{1}{2}x}$$

$$\Rightarrow y_p(x)=Ax^2 e^{\frac{1}{2}x}$$

$$y_p'=A\left[2x+\frac{1}{2}x^2\right]e^{\frac{1}{2}x}$$

$$y_p''=A\left[2+x+x+\frac{1}{4}x^2\right]e^{\frac{1}{2}x}$$

$$=A\left(\frac{1}{4}x^2+2x+2\right)e^{\frac{1}{2}x}$$

$$A(\cancel{x^2}\cancel{+8x}+8)+A(\cancel{-8x}\cancel{-2x^2})+\cancel{4x^2}=1$$

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$$8A=1 \Rightarrow A=\frac{1}{8}$$

$$\Rightarrow \quad y(x)=C_1e^{\frac{1}{2}x}+C_2xe^{\frac{1}{2}x}+\frac{1}{8}x^2e^{\frac{1}{2}x}$$

$$y'(x)=\frac{1}{2}C_1e^{\frac{1}{2}x}+C_2\left(1+\frac{1}{2}x\right)e^{\frac{1}{2}x}$$
$$+\frac{1}{8}\left(2x+\frac{1}{2}x^2\right)e^{\frac{1}{2}x}$$

$$\begin{cases} y(0)=-2=C_1 \\ y'(0)=1=\frac{1}{2}C_1+C_2 \end{cases} \Rightarrow \begin{cases} C_1=-2 \\ C_2=2 \end{cases}$$

$$\Rightarrow \cancel{y(x)=C_1e^{\frac{1}{2}x}+C_2xe^{\frac{1}{2}x}}$$

$$y(x)=-2e^{\frac{1}{2}x}+2xe^{\frac{1}{2}x}+\frac{1}{8}x^2e^{\frac{1}{2}x}$$
$$=\left(\frac{1}{8}x^2+2x-2\right)e^{\frac{1}{2}x}$$