SVOLGIMENTO PROVA SCRITTA di ANALISI 1 del 19/6/2014

A1

COMPITO A

1) $\cancel{x^2}-y^2+2ixy+2y=\cancel{x^2}+y^2+3i+2$

$$\Rightarrow \begin{cases} 2y^2-2y+2=0 & \leftarrow \Delta<0 \Rightarrow \nexists \text{ sol. } y\in\mathbb{R} \\ 2xy=3 \end{cases}$$

EQ. IMPOSSIBILE.

2) Criterio di Leibniz:

$$a_n = \operatorname{arctg}\left(\frac{1}{3n+2}\right) \xrightarrow[n\to\infty]{} 0$$

$$a_n = \operatorname{arctg}\left(\frac{1}{3n+2}\right) \geq a_{n+1} = \operatorname{arctg}\left(\frac{1}{3(n+1)+2}\right)$$

$$\Leftrightarrow \frac{1}{3n+2} \geq \frac{1}{3(n+1)+2}$$ (arctg(x) è MONOTONA CRESCENTE)

$$\Leftrightarrow 3n+2 \leq 3(n+1)+2$$ ovviamente vero $\forall n$.

La serie converge semplicemente;

la serie NON CONVERGE assolutamente:

$$\sum\left|(-1)^n \operatorname{arctg}\left(\frac{1}{3n+2}\right)\right| = \sum \operatorname{arctg}\left(\frac{1}{3n+2}\right)$$

$$\approx \sum \frac{1}{3n+2}$$ divergente.

A2

3) Per $x \to 0$

$$f(x) \sim \frac{1}{\log(1+x)} \sim \frac{1}{x} \quad \text{NON INTEGRABILE.}$$

Poichè in $(0,1]$ $f(x) \geq 0$, allora

$$\int_0^1 f(x) = +\infty.$$

Anche se non ~~richiesto~~ NECESSARIO, calcoliamo esplicitamente l'integrale, ponendo $\log(1+x) = t$ $dt = \frac{1}{1+x} dx$

$t(0) = 0$; $t(1) = \log 2$

$$\Rightarrow \int_0^{\log 2} \frac{1}{(t^2+1)t} dt$$

Sfruttando i fratti semplici, si riscrive l'integrando nella forma

$$\frac{1}{(t^2+1)t} = \frac{At+B}{t^2+1} + \frac{C}{t} = \frac{At^2+Bt+Ct^2+C}{(t^2+1)t}$$

da cui $\begin{cases} A = -C = -1 \\ B = 0 \\ C = 1 \end{cases}$

$$\Rightarrow \int_0^{\log 2} \left[\frac{-t}{t^2+1} + \frac{1}{t}\right] dt = \cancel{\left[\log t - \operatorname{arctg} t\right]_0^{\log 2}}$$

$$= \log t - \frac{1}{2}\log(t^2+1)\Big|_0^{\log 2} = \log\left[\frac{t}{\sqrt{t^2+1}}\right]_0^{\log 2} =$$

$$= \log\left[\frac{\log 2}{\sqrt{\log^2 2+1}}\right] - \lim_{x\to 0^+} \log\left[\frac{t}{\sqrt{t^2+1}}\right] = +\infty.$$

A3

4)
$$\lim_{x\to 0} \frac{\left[\cancel{1}-\cancel{\frac{x^2}{2}}+\frac{1}{2}\left(-\frac{x^2}{2}\right)^2+o(x^4) - \left[\cancel{1}-\cancel{\frac{x^2}{2}}+\frac{x^4}{4!}+o(x^4)\right]\right]x^2}{\left[\cancel{x^2}-\frac{x^6}{6}+o(x^6)\right]-\left[\cancel{x^2}+\frac{x^6}{6}+o(x^6)\right]} =$$

$$= \lim_{x\to 0} \frac{\left(\frac{x^4}{8}-\frac{x^4}{24}+o(x^4)\right)x^2}{-\frac{x^6}{3}+o(x^6)} = \lim_{x\to 0} \frac{\frac{1}{12}x^6}{-\frac{x^6}{3}} = -\frac{1}{4}.$$

5) $I_{def} = \{x \neq -3\}$ $\qquad f(x) \geq 0 \quad \forall x \in I_{def}$

NO INTERSEZIONI CON ASSE x.

$$f(x) = \begin{cases} \dfrac{x^2+4}{x+3} & \text{se } x > -3 \\ -\left(\dfrac{x^2+4}{x+3}\right) & \text{se } x < -3 \end{cases}$$

$f(0) = \frac{4}{3}$

$$f'(x) = \begin{cases} \dfrac{2x(x+3)-(x^2+4)}{(x+3)^2} = \dfrac{x^2+6x-4}{(x+3)^2} & \text{per } x > -3 \\ -\left[\dfrac{x^2+6x-4}{(x+3)^2}\right] & \text{per } x < -3 \end{cases}$$

$x^2+6x-4 > 0$ per $x < \frac{-3-\sqrt{13}}{}$; $x > -3+\sqrt{13}$

f' è negativa in $(-\infty, -3-\sqrt{13})$; positiva in $(-3-\sqrt{13}, -3)$; negativa in $(-3, -3+\sqrt{13})$; positiva in $(-3+\sqrt{13}, +\infty)$.

$x_1 = -3-\sqrt{13}$ punto di MIN. REL.

$$f(x_1) = \frac{9+13+6\sqrt{13}+4}{\sqrt{13}} = \frac{26+6\sqrt{13}}{\sqrt{13}} = 2\sqrt{13}+6$$

$x_2 = -3+\sqrt{13}$ punto di MIN. REL.

$$f(x_2) = \frac{9+13-6\sqrt{13}+4}{\sqrt{13}} \quad \frac{26-6\sqrt{3}}{\sqrt{13}} \Leftarrow \underline{\text{MIN. ASS.}}$$

$= 2\sqrt{13}-6 \cong 1{,}2 < \frac{4}{3}$

$$\lim_{x\to -3^+} f(x) = \frac{13}{0^+} = +\infty \ ; \ \lim_{x\to -3^-} = \frac{13}{0^+} = +\infty$$

$x=-3$ AS. VERTICALE DX e SX

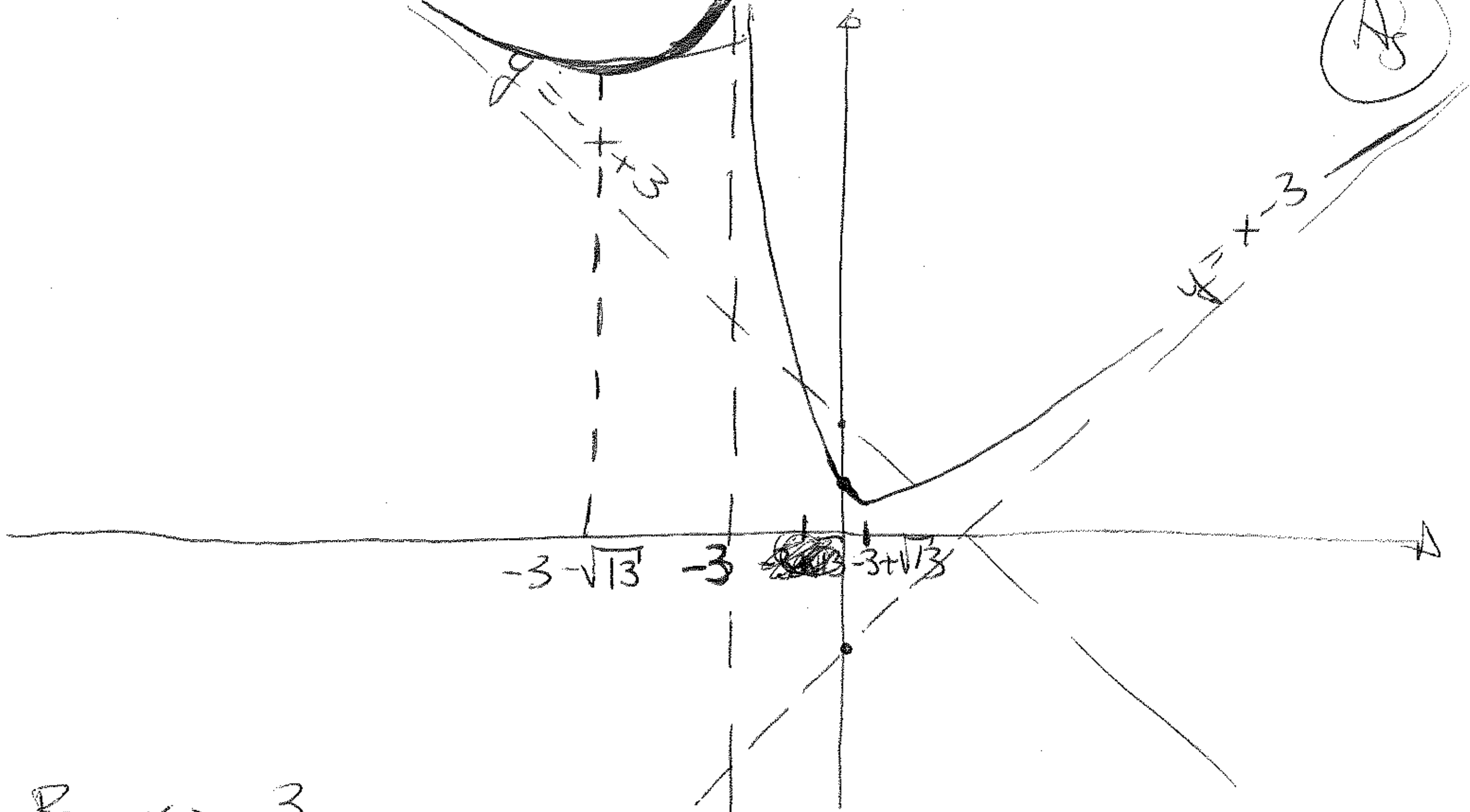
$$\lim_{x\to \pm\infty} f(x) = +\infty$$

per $x>-3$

$$f(x) = \frac{x^2+3x-3x-\cancel{9}+13}{x+3} = x-3+\underbrace{\frac{13}{x+3}}_{\xrightarrow{+\infty} 0}$$

$\Rightarrow y = x-3$ AS. OBLIQUO per $x\to+\infty$

$$f(x) \underset{x\to-\infty}{\sim} -x+3 \quad \Rightarrow \quad y=-x+3 \text{ AS. OBLIQUO per } x\to-\infty$$



Per $x > -3$

$$f''(x) = \frac{(2x+6)(x+3)^{\cancel{2}} - (x^2+6x-4)\,2\cancel{(x+3)}}{(x+3)^3} = \frac{\cancel{2x^2}+\cancel{12x}+18-\cancel{2x^2}-\cancel{12x}+8}{(x+3)^3} = \frac{26}{(x+3)^3} > 0$$

$$\Rightarrow \quad f''(x) = \begin{cases} \dfrac{26}{(x+3)^3} > 0 & \text{se } x > -3 \\[2ex] \dfrac{-26}{(x+3)^3} > 0 & \text{se } x < -3 \end{cases}$$

f convessa in $(-\infty, -3)$ e in $(-3, +\infty)$.