SVOLGIMENTI DI ANALISI MAT. 1 del. 20/1/2017

A, 1

COMPITO A

1) $f(x)=\dfrac{1}{e^{2x}-1} \in C^0\big((-\infty,0)\cup(0,+\infty)\big)$.

Poiché $x_0=-\log 2<0 \Rightarrow$ il Pb. di Cauchy è definito in $(-\infty,0)$.

$\exists !$ soluzione $y \in C^1(-\infty,0)$. Soluzione GLOBALE.

A_{1bis}

$$y(x) = e^{-\int_{-\log 2}^{x} 1\,dt} \left[\int_{-\log 2}^{x} e^{\int_{-\log 2}^{t} 1\,ds} \frac{1}{e^{2t}-1}\, dt - \log 3 \right]$$

$$= e^{-x-\log 2}\left[\int_{-\log 2}^{x} e^{t+\log 2}\,\frac{1}{e^{2t-1}}\, dt - \log 3\right]$$

$$= \frac{1}{2} e^{-x}\left[2\int_{-\log 2}^{x} \frac{e^t}{e^{2t}-1}\,dt - \log 3\right]$$

$u = e^t \Rightarrow du = e^t\,dt$; $u(-\log 2) = \frac{1}{2}$; $u(x) = e^x$

$$\Rightarrow \quad = e^{-x}\left[\int_{1/2}^{e^x}\frac{du}{u^2-1}\, du - \frac{1}{2} \log 3\right]$$

$$= e^{-x}\left[\frac{1}{2}\int_{1/2}^{e^x}\left[\frac{-1}{u+1}+\frac{1}{u-1}\right]du - \frac{1}{2}\log 3\right]$$

A2

$$= e^{-x}\left[\frac{1}{2}\log\left|\frac{u-1}{u+1}\right|\Bigg|_{\frac{1}{2}}^{e^x} - \frac{1}{2}\log 3\right]$$

$$= e^{-x}\left[\frac{1}{2}\log\left(\left|\frac{e^x-1}{e^x+1}\right|\cdot\left|\frac{\frac{3}{2}}{-\frac{1}{2}}\right|\right) - \frac{1}{2}\log 3\right]$$

$$= e^{-x}\left[\frac{1}{2}\log\left(3\left(\frac{1-e^x}{e^x+1}\right)\right) - \frac{1}{2}\log 3\right].$$

si ricordi che $x<0$

2) ~~Per~~ $\begin{cases} \log(1+x^3-x^4)>0 \\ 1+x^3-x^4>0 \end{cases}$

$$\Rightarrow \begin{cases} 1+x^3-x^4>1 \\ \cancel{1+x^3-x^4>0} \end{cases} \leftarrow \quad \begin{array}{l} x^3-x^4>0 \\ x^3(1-x)>0 \end{array}$$

$\Rightarrow$ Per $\underbrace{0<x<1}$ la funzione è continua.

Per $x \to 0$ $\quad f(x) \sim \dfrac{x}{\sqrt{x^3+o(x^3)}} = \dfrac{1}{\sqrt{x}}$

~~NON~~ INTEGRABILE.

A_3

3) $z^2+\bar{z} = x^2-y^2+x+i(2xy-y)$

$\overline{z^2+\bar{z}} = x^2-y^2+x+i(-2xy+y)$

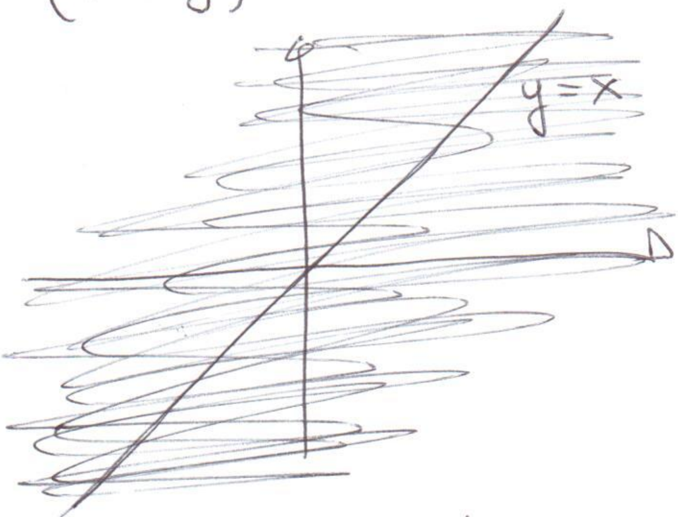
$Im\left(\overline{z^2+\bar{z}}\right) = -2xy+y$

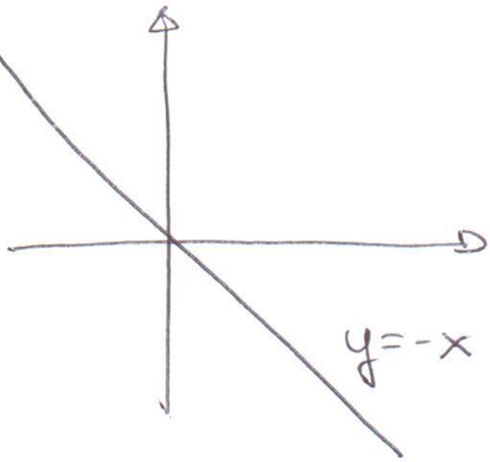
$|z|^2-iz = x^2+y^2+y-ix$

$Re\left(|z|^2-iz\right) = x^2+y^2+y$

$\Rightarrow \quad -2xy+\cancel{y} = x^2+y^2+\cancel{y}$

$\Rightarrow \quad (x+y)^2 = 0 \quad \Rightarrow \quad y=-x$





4) Criterio della radice:

$$\sqrt[n]{a_n} = \frac{n}{(n+1)}\cdot\frac{1}{\left(1+\frac{1}{n}\right)^n} \longrightarrow \frac{1}{e}<1$$

Serie convergente.

5) $D = \{x \neq -1\}$ A_4

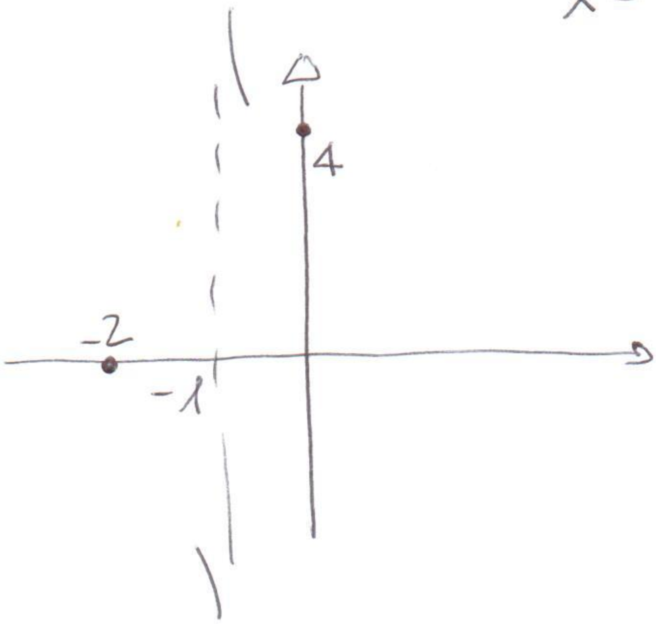
$f(x) = 0 \iff x = -2$

$f(x) \geq 0 \iff x > -1$

$f(x) \leq 0 \iff x < -1$

$\lim\limits_{x \to -1^{\pm}} f(x) = \dfrac{1}{0^{\pm}} = \pm\infty$ AS. VERTICALE DX e SX $x = -1$

$f(0) = 4$



$$f(x) = \frac{x^2+4x+4}{x+1}$$

$$= \frac{x^2+x+3x+3+1}{x+1}$$

$$= x + 3 + \frac{1}{x+1} \implies$$ AS. OBLIQUO $y = x+3$ per $x \to \pm\infty$.

$$f'(x) = 1 - \frac{1}{(x+1)^2} = \frac{(x+1)^2 - 1}{(x+1)^2} = 0 \iff (x+1)^2 = 1$$

$\iff x = 0 \ ; \ x = -2.$

$f'(x) > 0 \iff (x+1)^2 > 1 \iff x < -2 \ ; \ x > 0$

A_5

f cresce in $(-\infty,-2)$; decresce in $(-2,-1)$; decresce in $(-1,0)$; cresce in $(0,+\infty)$.

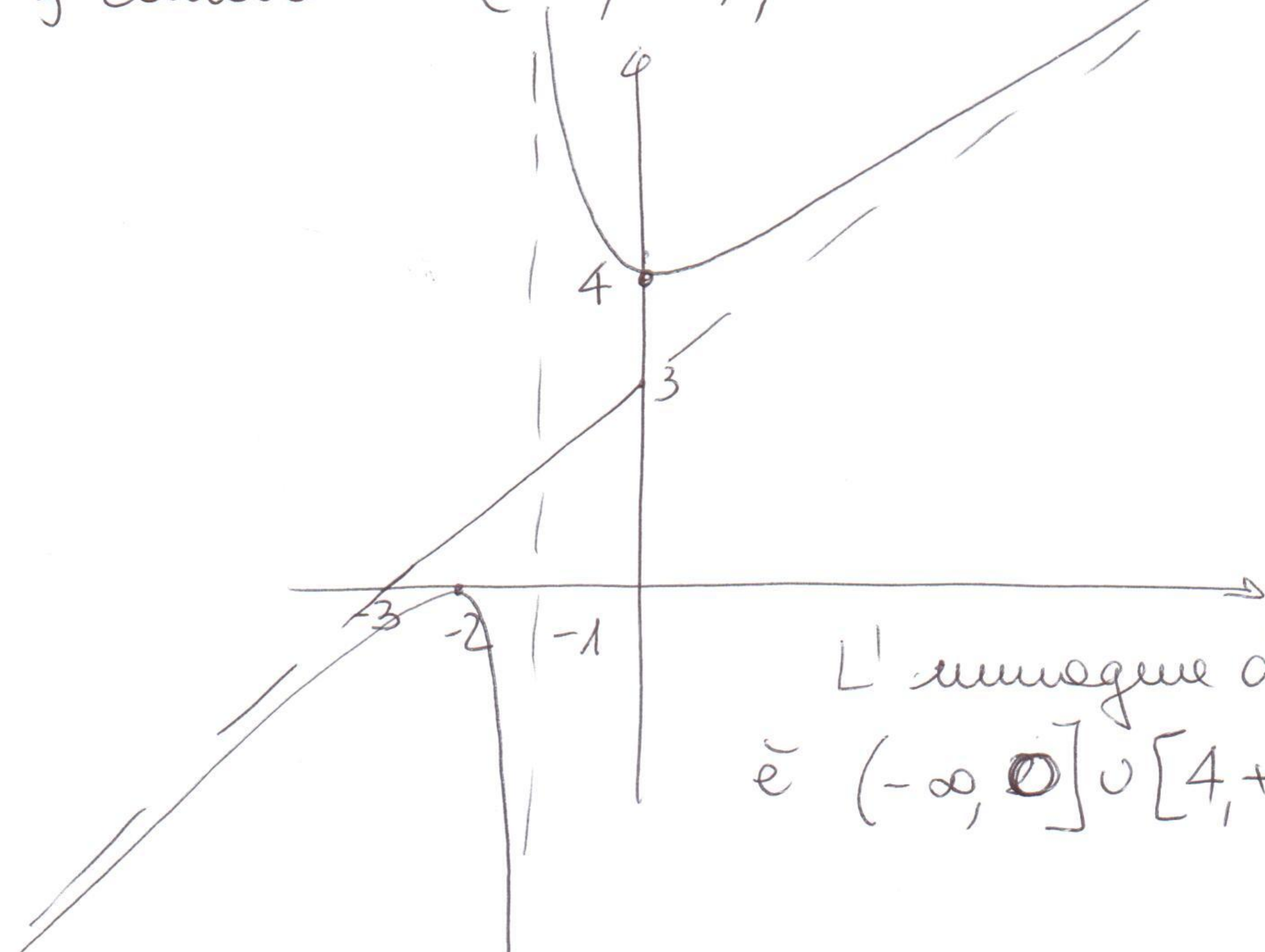
$x_1=-2$ punto di MAX. REL.

$x_2=0$ punto di MIN. REL.

Poichè $f(x) \xrightarrow[x\to\pm\infty]{} \pm\infty$ $\nexists$ MAX-MIN. ASS.

$$f''(x)=\frac{2}{(x+1)^3}>0 \iff x>-1$$

f concava in $(-\infty,-1)$; convessa in $(-1,+\infty)$.



L'immagine di f è $(-\infty,0]\cup[4,+\infty)$.