COMPITO B

B1

1) $$z = \frac{\left[\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)\right]^6}{2\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]} =$$

$$= \frac{8\left[\cos\left(\frac{\pi}{4}\right) + i \operatorname{sen}\left(\frac{\pi}{4}\right)\right]^6}{2\left[\cos\left(\frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{3}\right)\right]} =$$

$$= 4\,\frac{\left[\cos\left(\frac{3}{2}\pi\right) + i \operatorname{sen}\left(\frac{3}{2}\pi\right)\right]}{\left[\cos\left(\frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{3}\right)\right]}$$

$$= 4\left[\cos\left(\frac{3}{2}\pi - \frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{3}{2}\pi - \frac{\pi}{3}\right)\right]$$

$$= 4\left[\cos\left(\frac{7}{6}\pi\right) + i \operatorname{sen}\left(\frac{7}{6}\pi\right)\right] = 4\left[-\frac{\sqrt{3}}{2} - i\,\frac{1}{2}\right]$$

$$= -2\sqrt{3} - 2i$$

2) $D = \mathbb{R}$ $\quad f \in C^0(\mathbb{R})$ NO ASINTOTI VERTICALI

$f(x) = 0 \iff x = 2$

(B_2)

$f(x) > 0 \iff x^3 - 8 > 0 \iff x > 2$

$$f(x) = x\left(1 - \frac{8}{x^3}\right)^{\frac{1}{3}} \underset{x \to \pm\infty}{\frown} x\left(1 - \frac{8}{3x^3}\right)$$

$$= x - \frac{8}{3x^2} = x + o(1)$$

$\Rightarrow$ $y = x$ è AS. OBLIQUO a $\pm\infty$.

$$f'(x) = \frac{1}{3}\left(x^3 - 8\right)^{-\frac{2}{3}} \cdot 3x^2 = \frac{x^2}{\left(x^3 - 8\right)^{2/3}}$$

NON DEFINITA in $x = 2$

$$\lim_{x \to 2^\pm} f'(x) = \frac{4}{0^+} = +\infty$$

flesso a tangente verticale.

$f'(x) \geq 0 \quad \forall x \neq 2$

$f'(x) = 0 \iff x = 0$

f sempre crescente.

$x = 0$ punto di flesso orizzontale

$f(0) = -2$

$$f''(x) = \frac{2x\,(x^3-8)^{2/3} - x^2\,\frac{2}{\cancel{3}}\,(x^3-8)^{-1/3}\cdot \cancel{3}x^2}{(x^3-8)^{4/3}}$$

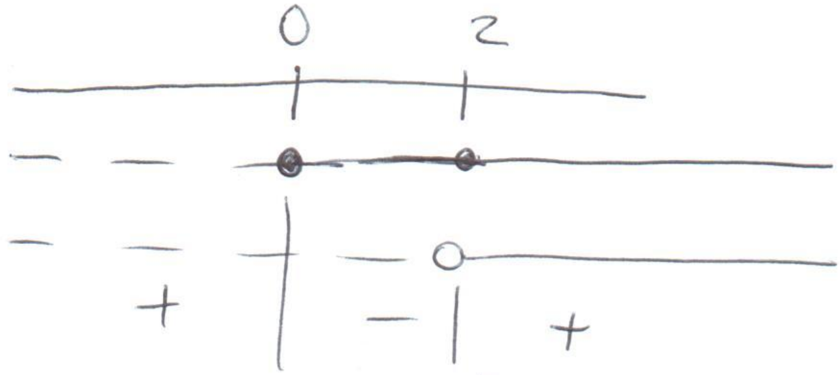
B_3

$$= \frac{2x\,(x^3-8) - 2x^4}{(x^3-8)^{5/3}} = \frac{-16x}{(x^3-8)^{5/3}} = 0$$

$\iff$ $x = 0$ (già verificato)

$$f''(x) > 0 \iff \frac{x}{(x^3-8)^{5/3}} < 0$$

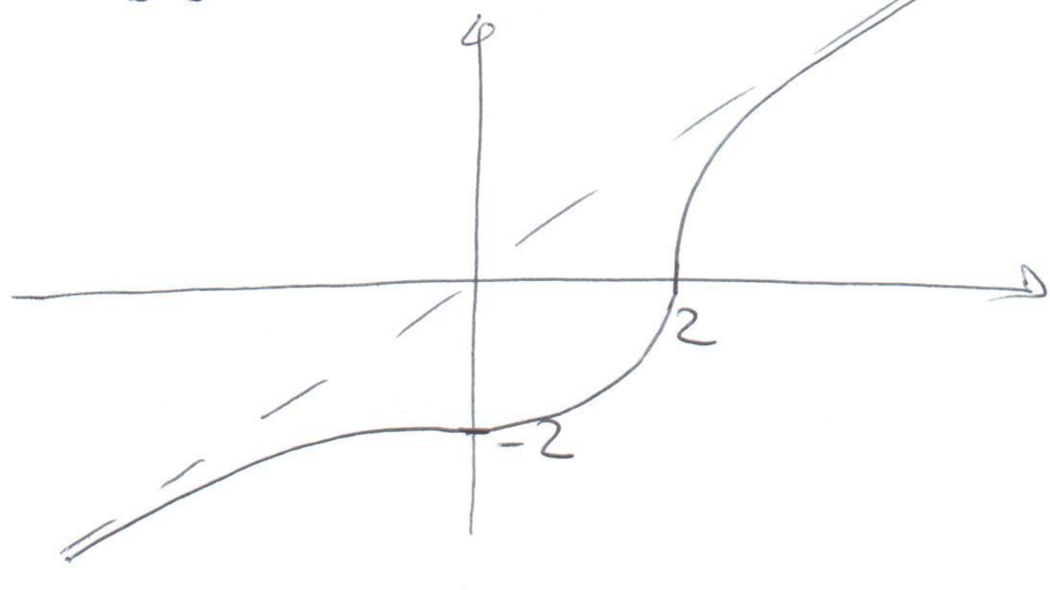
$\iff$



CONCAVA CONV. CONCAVA

flesso orizzontale discendente

flesso verticale discendente



B4

3)

$$y(x) = e^{-\int \cos x \, dx}\left[\int e^{\int \cos x \, dx} \cos^3 x \, dx + C\right]$$

$$= e^{-\operatorname{sen} x}\left[\int e^{\sin x}(1-\sin^2 x)\cos x \, dx + C\right]$$

$$t = \sin x \implies dt = \cos x \, dx$$

$$= e^{-\operatorname{sen} x}\left[C + \int (1-t^2)e^t \, dt\right]_{t=\sin x}$$

$$\boxed{\begin{aligned}\int (1-t^2)e^t \, dt &= e^t - \left[t^2 e^t - \int 2te^t \, dt\right] \\ &= e^t - t^2 e^t + 2te^t - 2\int e^t \, dt = \\ &= (-t^2+2t-1)e^t + c = -(t-1)^2 e^t + c\end{aligned}}$$

$$= e^{-\operatorname{sen} x}\left[C - (\sin x - 1)^2 e^{\operatorname{sen} x}\right]$$

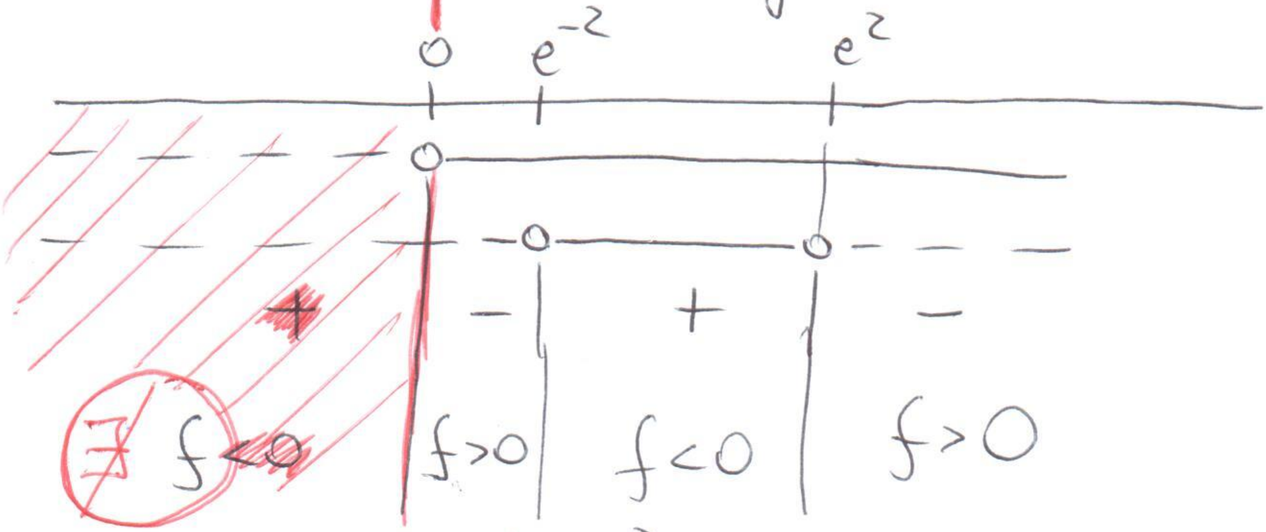
$$= -(\operatorname{sen} x - 1)^2 + Ce^{-\sin x}$$

(B5)

Poichè
$$e^{-1} \leq e^{-\sin x} \leq e^{1}$$
e
$$|\sin x - 1|^2 \leq 2^2 = 4$$
TUTTE le soluzioni sono limitate.

4) $f(x) > 0$: Studio $\dfrac{1}{x(4-\log^2 x)}$

$$4 - \log^2 x > 0 \iff \log^2 x < 4$$
$$\iff -2 < \log x < 2 \qquad e^{-2} < x < e^2$$



Poichè $e^3, e^4 > e^2 \Rightarrow f(x) > 0$ in $[e^3, e^4]$

$$\text{Area} = \int_{e^3}^{e^4} \frac{-1}{x(4-\log^2 x)}\,dx$$

$t = \log x$; $dt = \frac{1}{x}dx$; $t(e^3) = 3$; $t(e^4) = 4$

$$= -\int_3^4 \frac{1}{(4-t^2)}\,dt = \int_3^4 \frac{1}{(t-2)(t+2)}\,dt$$

$$\frac{1}{4}\int_3^4 \left[\frac{1}{t-2} - \frac{1}{t+2}\right] dt$$

$$= \frac{1}{4} \log\left|\frac{t-2}{t+2}\right| \Bigg|_3^4 = \frac{1}{4}\left[\log\left(\frac{2}{6}\right) - \log\left(\frac{1}{5}\right)\right]$$

$$= \frac{1}{4}\log\left(\frac{5}{3}\right).$$

5) $$\sum \frac{k\log(k)}{k^\alpha} = \sum \frac{1}{[\log(k)]^{-1} k^{\alpha-1}}$$ Serie di Abel

Se $\alpha > 2$ converge

Se $\alpha < 2$ diverge

Se $\alpha = 2$ $\quad \sum \frac{1}{[\log k]^{-1} k}$ diverge