COMPITO C

C_1

1) Serie a termini positivi

$$n^2 \operatorname{arctg}(n)\left[e^{1/n^2}-1-\frac{1}{n^2}\right]$$ criterio del confronto

$$\leq n^2\frac{\pi}{2}\left(e^{1/n^2}-1-\frac{1}{n^2}\right)$$ confronto asintotico

$$\sim \frac{\pi}{2}n^2\frac{1}{2n^4}=\frac{\pi}{4n^2}.$$

Poichè $\sum \frac{\pi}{4n^2}$ converge, allora converge anche la serie proposta.

2) Per $x\to+\infty$

$$e^{1/\sqrt{x}}-1\sim\frac{1}{\sqrt{x}} \Rightarrow \sinh\left(e^{1/\sqrt{x}}-1\right)\sim\frac{1}{\sqrt{x}}$$

$$e^{\frac{1}{\sqrt{x}}}\longrightarrow e^{\frac{1}{\infty}}=e^0=1$$

$$\Rightarrow f(x)\underset{x\to+\infty}{\sim}\frac{1}{2x^{3/2}}\cdot\frac{1}{\sqrt{x}}=\frac{1}{2x^2}$$

che è integrabile.

C_2

Effettuiamo la sostituzione

$$e^{1/\sqrt{x}} - 1 = t \quad \Rightarrow \quad dt = \frac{-1}{2x^{3/2}} e^{\left(x^{-\frac{1}{2}}\right)} dx$$

$$t(1) = e-1 \; ; \quad t(+\infty) = 0$$

$$\Rightarrow \int_1^{+\infty} f(x)\,dx = -\int_{e-1}^{0} \sinh(t)\,dt$$

$$= -\cosh(t)\Big|_{e-1}^{0} = -1 + \cosh(e-1).$$

3) omo.: $\alpha^2 - 2\alpha = 0 \Rightarrow \alpha(\alpha - 2) = 0$

$$\Rightarrow y_0(x) = c_1 + c_2 e^{2x}$$

Poiché $\alpha = 0$ è radice del polinomio caratteristico $\Rightarrow \quad w(x) = Ax$;

$$w'(x) = A; \quad w''(x) = 0$$

$$-2A = \frac{3}{2} \quad \Rightarrow \quad A = -\frac{3}{4}.$$

$$\Rightarrow y_{NO}(x) = c_1 + c_2 e^{2x} - \frac{3}{4}x.$$

$$y(0) = C_1 + C_2 = 0$$

$$y'(x) = 2C_2 e^{2x} - \frac{3}{4} \Rightarrow y'(0) = 2C_2 - \frac{3}{4} = 1$$

$$\Rightarrow \begin{cases} C_2 = \dfrac{7}{8} \\ C_1 = -\dfrac{7}{8} \end{cases}$$

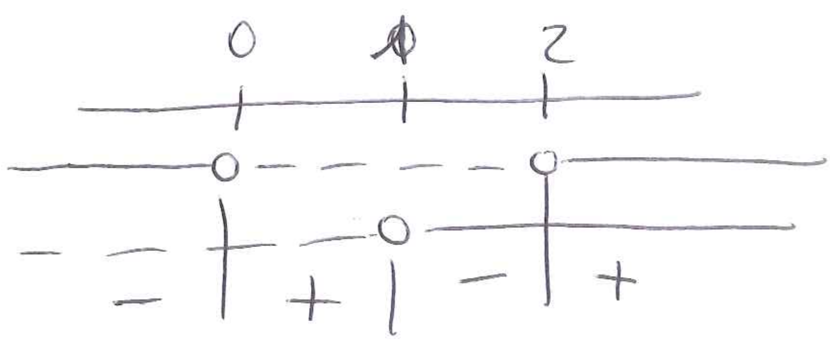
$$\Rightarrow \underline{y(x) = -\frac{7}{8} + \frac{7}{8} e^{2x} - \frac{3}{4} x.}$$

4) $$f(x) = \frac{\log\left[(x-1)^2\right]}{x-1} = \frac{2\log(|x-1|)}{x-1}$$

$$D = \{x \in \mathbb{R} \mid x \neq 1\} = (-\infty, 1) \cup (1, +\infty).$$

Segno: $\begin{cases} \log(|x-1|) > 0 \\ x > 1 \end{cases} \Rightarrow \begin{cases} |x-1| > 1 \\ x > 1 \end{cases}$

$$\Rightarrow \begin{cases} x < 0 \,;\; x > 2 \\ x > 1 \end{cases}$$



$$f(0) = 0$$

$$f(x) = 0 \iff \log(|x-1|) = 0 \iff |x-1| = 1$$

$$\iff x = 0 \,;\; x = 2.$$

$$\lim_{x \to \pm\infty} f(x) = \frac{2\log(+\infty)}{\pm\infty} = 0^{\pm}$$

$y=0$ AS. ORIZZ. a $\pm\infty$

C4

$$\left(\log|x-1| \ll |x-1|\right)$$

$$\lim_{x \to 1^{\pm}} f(x) = \frac{2\log(0^+)}{0^{\pm}} = \mp\infty.$$

$x=1$ AS. VERT.

$$f(x) = \begin{cases} \dfrac{2\log(x-1)}{x-1} & \text{se } x>1 \\[2ex] \dfrac{2\log(1-x)}{x-1} & \text{se } x<1 \end{cases}$$

$$f'(x) = \begin{cases} 2\left[\dfrac{1-\log(x-1)}{(x-1)^2}\right] & \text{se } x>1 \\[2ex] 2\left[\dfrac{1-\log(1-x)}{(x-1)^2}\right] & \text{se } x<1 \end{cases}$$

<u>Per $x>1$</u> $\quad f'(x)>0 \iff 1-\log(x-1)>0$

$\iff \log(x-1)<1 \iff \underline{x<e+1}$

$$f(e+1) = \frac{2}{e}$$

Per $x<1$ $\quad f'(x)>0 \iff 1-\log(1-x)>0$

$\iff \log(1-x)<1 \iff x>1-e$

C5

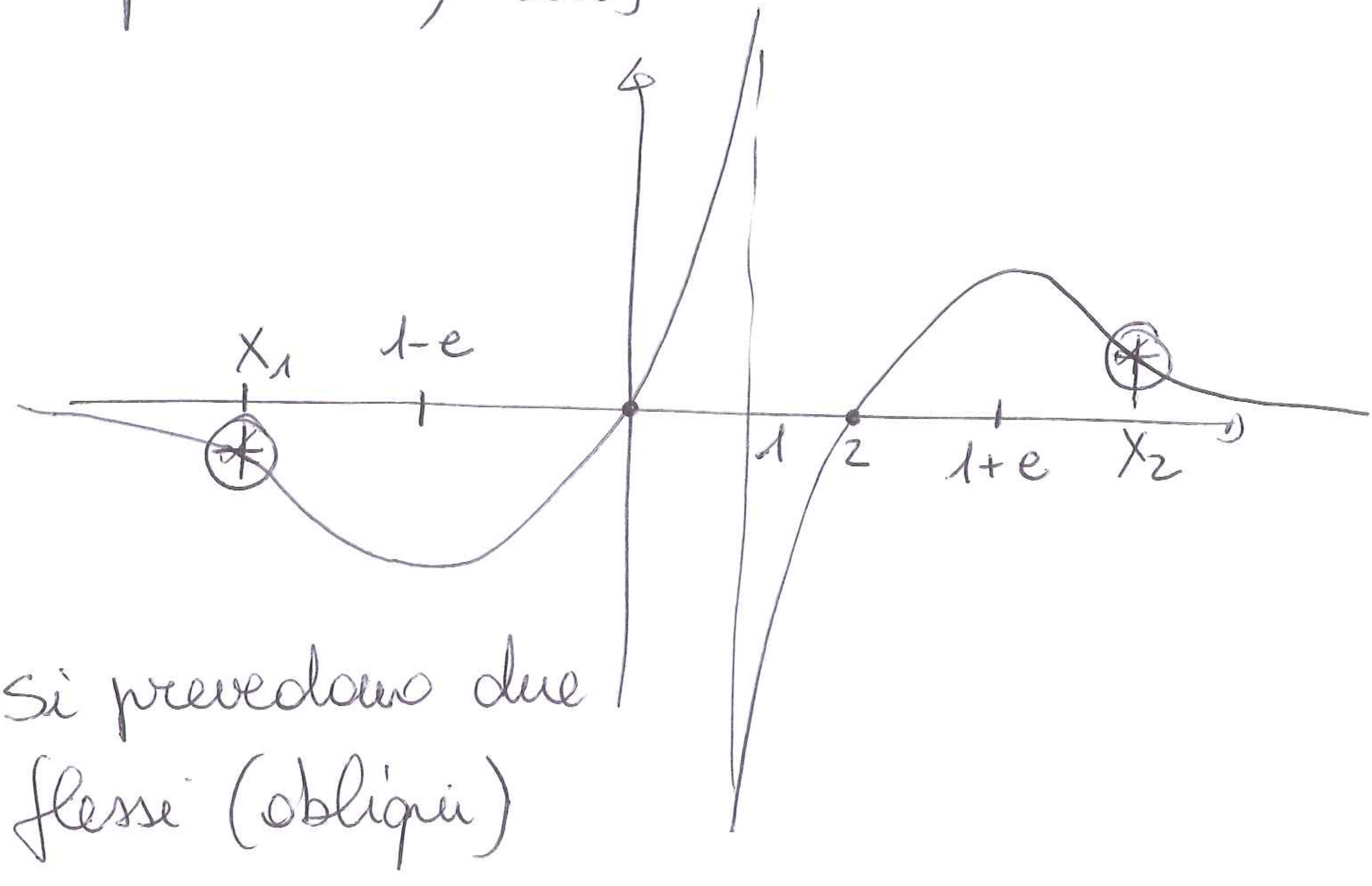
$$f(1-e) = -\frac{2}{e}$$

f decresce in $(-\infty, 1-e)$; cresce in $\left(-\frac{2}{e}, 0\right)$; cresce in $(0, 1+e)$; decresce in $(1+e, +\infty)$.

$x = 1+e$ punto di MAX. REL.

$x = 1-e$ punto di MIN. REL.

$\sup = +\infty$; $\inf = -\infty$.



Si prevedono due flessi (obliqui)

N.B.: per $x>1$ 6

$$f''(x)=2\left[\frac{-\frac{1}{\cancel{(x-1)}}\cancel{(x-1)^2}-(1-\log(x-1))\,2\,\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}}\right]$$

$$=2\left[\frac{-3+2\log(x-1)}{(x-1)^3}\right]>0$$

$$\iff \quad 2\log(x-1)>3 \quad \iff \quad x>1+e^{3/2}$$

Per $x<1$:

$$f''(x)=2\left[\frac{-\frac{1}{\cancel{(x-1)}}\cancel{(x-1)^2}-(1-\log(1-x))\,2\,\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}}\right]$$

$$=\frac{2}{(x-1)^3}\left[-3+2\log(1-x)\right]$$

$(x-1)^3<0$ per $x<1$

$$\Rightarrow \quad f''(x)>0 \quad \iff \quad 2\log(1-x)<3$$

$$\iff \quad x>1-e^{3/2}$$

f è CONCAVA in $(-\infty,\, 1-e^{3/2})$;
CONVESSA in $(1-e^{3/2},\, 1)$;
~~CONVES~~ CONCAVA in $(1,\, 1+e^{3/2})$;
CONVESSA in $(1+e^{3/2},\, +\infty)$

In $x_1 = 1 - e^{3/2}$ e in $x_2 = 1 + e^{3/2}$ (C7)

vi sono due flessi.

5) $Re(z) = Re(\bar{z}) = x$

$$z^2 = x^2 - y^2 + 2ixy \Rightarrow iz^2 = -2xy + i(x^2 - y^2)$$

$$\Rightarrow Re(iz^2) = -2xy$$

$$\Rightarrow e^{x + \cancel{\frac{2xy}{x}} i} = 2\sqrt{2}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right] \qquad (x \neq 0)$$

$$\Rightarrow e^x \cdot e^{+2yi} = 2\sqrt{2} \cdot e^{-\frac{\pi}{4} i}$$

Per la periodicità dell'esponenziale immaginario, si ha

$$\begin{cases} e^x = 2\sqrt{2} \\ +2y = -\frac{\pi}{4} + 2k\pi \end{cases} ; k \in \mathbb{Z}$$

$$\begin{cases} x = \log(2\sqrt{2}) = \frac{3}{2} \log 2 \\ y = -\frac{\pi}{8} + k\pi \end{cases} ; k \in \mathbb{Z}$$