SVOLGIMENTO PROVA di ANALISI 1
del 22/2/2013

COMPITO A

A_0

1) $z \neq i$

$$z - 2 + \frac{3}{2} i - z(z - i) = 0$$

$$z^2 - (i+1) z + 2 - \frac{3}{2} i = 0$$

$$z_{1,2} = \frac{(i+1) + \sqrt{(i+1)^2 - 8 + 6i}}{2} = \frac{(i+1) + \sqrt{8i - 8}}{2}$$

Poiché $|8i - 8| = 8\sqrt{2}$

$$\Rightarrow 8i - 8 = 8\sqrt{2}\left[-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}\right]$$

$$= 8\sqrt{2}\left[\cos\left(\frac{3}{4}\pi\right) + i \sin\left(\frac{3}{4}\pi\right)\right]$$

$$\sqrt{8i - 8} = \begin{cases} \sqrt[4]{128}\left[\cos\left(\frac{3}{8}\pi\right) + i \sin\left(\frac{3}{8}\pi\right)\right] \\ \sqrt[4]{128}\left[\cos\left(\frac{11\pi}{8}\right) + i \sin\left(\frac{11}{8}\pi\right)\right] \end{cases}$$

$$= \pm \sqrt[4]{128}\left[\cos\left(\frac{3}{8}\pi\right) + i \sin\left(\frac{3}{8}\pi\right)\right]$$

$$\Rightarrow z_{1,2} = \frac{(i+1) \pm \sqrt[4]{128}\left[\cos\left(\frac{3}{8}\pi\right) + i \sin\left(\frac{3}{8}\pi\right)\right]}{2}$$

Tramite le formule di bisezione A_1

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos x}{2}},$$

tenendo conto che $\alpha = \frac{3}{8}\pi$ si trova nel primo quadrante, si ha

$$\cos\left(\frac{3}{8}\pi\right) = \sqrt{\frac{1+\cos\left(\frac{3}{4}\pi\right)}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$= \frac{1}{2}\sqrt{2-\sqrt{2}}$$

$$\sin\left(\frac{3}{8}\pi\right) = \sqrt{\frac{1-\cos\left(\frac{3}{4}\pi\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

$$= \frac{1}{2}\sqrt{2+\sqrt{2}}$$

da cui

$$z_{1,2} = \frac{(i+1) \pm \sqrt[4]{128}\cdot\frac{1}{2}\left[\sqrt{2-\sqrt{2}} + i\sqrt{2+\sqrt{2}}\right]}{2}$$

$$= \frac{(i+1) \pm 2^{3/4}\left[\sqrt{2-\sqrt{2}} + i\sqrt{2+\sqrt{2}}\right]}{2}$$

2) $$\frac{-5x+4}{(x^2+3x-4)(x+4)} = \frac{-5x+4}{(x+4)^2(x-1)}$$

A_2

$$= \frac{A}{(x+4)} + \frac{B}{(x+4)^2} + \frac{C}{(x-1)} = \frac{A(x+4)(x-1)+B(x-1)+C(x+4)^2}{(x+4)^2(x-1)}$$

$$= \frac{(A+C)x^2+(3A+B+8C)x-4A-B+16C}{(x+4)^2(x-1)}$$

$$\begin{cases} A+C=0 \\ 3A+B+8C=-5 \\ -4A-B+16C=4 \end{cases} \qquad \begin{cases} C=-A \\ -5A+B=-5 \\ -20A-B=4 \end{cases} \qquad \begin{cases} C=-A \\ -25A=-1 \\ B=-5+5A \end{cases}$$

$$\begin{cases} C=-A=-\frac{1}{25} \\ A=\frac{1}{25} \\ B=-5+\frac{1}{5}=-\frac{24}{5} \end{cases}$$

$$\int_2^{+\infty} f(x)\,dx = \frac{1}{25}\int_2^{+\infty} \left[\frac{1}{(x+4)} - \frac{120}{(x+4)^2} - \frac{1}{(x-1)}\right]dx$$

$$= \frac{1}{25}\left[\log\left(\frac{x+4}{x-1}\right) + \frac{120}{x+4}\right]_2^{+\infty} =$$

$$= \frac{1}{25}\left\{\lim_{x\to+\infty}\left[\log\left(\frac{x+4}{x-1}\right) + \frac{120}{x+4}\right] - \log 6 - \frac{12}{5}\right]$$

$$= -\frac{1}{25}\left(\log 6 + \frac{12}{5}\right).$$

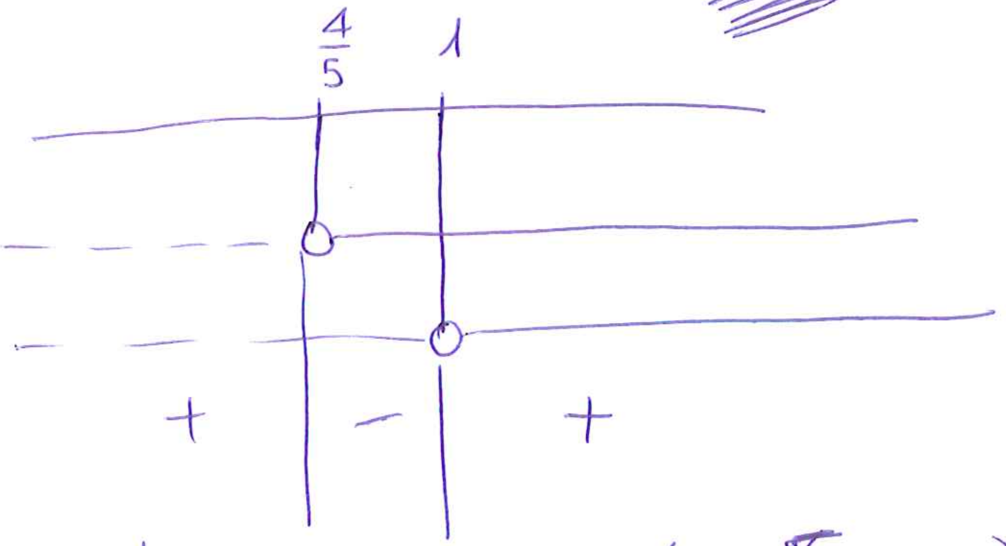
A3

In $[2,+\infty)$

~~quindi~~

$$f(x)=\frac{-5x+4}{(x+4)^2(x-1)}>0$$

~~cioè~~ $$\iff \frac{5x-4}{x-1}<0$$



quindi $f(x)<0 \quad \forall x\in[2,+\infty)$.

f è sicuramente integrabile in $[2,+\infty)$, perchè, per $x\to+\infty$

$$f(x)\sim\frac{-5x}{x^3}=\frac{-5}{x^2}$$

che è integrabile in $[2,+\infty)$.

3) $f(x) = e \cdot e^{-2|\log(x+1)|} \geq 0 \quad \forall x \in I_{def}$ (A4)

I_{def}: $x+1>0 \Rightarrow x>-1$

$$I_{def} = (-1, +\infty)$$

$$|\log(x+1)| = \begin{cases} \log(x+1) & \text{se } x+1 \geq 1 \\ -\log(x+1) & \text{se } x+1<1 \end{cases}$$

$$= \begin{cases} \log(x+1) & \text{se } x \geq 0 \\ -\log(x+1) & \text{se } -1<x<0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} e \cdot e^{2\log(x+1)} = e(x+1)^2 & \text{se } -1<x<0 \\ e \cdot e^{-2\log(x+1)} = \dfrac{e}{(x+1)^2} & \text{se } x \geq 0 \end{cases}$$

$\lim_{x \to -1^+} f(x) = 0$, f è prolungabile per continuità fino in $x=-1$.
NO AS. VERTICALI

$\lim_{x \to +\infty} f(x) = 0$

AS. ORIZZONTALE a $+\infty$:

$$y=0$$

$$f'(x) = \begin{cases} 2e(x+1)>0 & \text{se } -1<x<0 \\ \dfrac{-2e}{(x+1)^3}<0 & \text{se } x>0 \end{cases}$$

$$\lim_{x\to 0^+} f'(x) = -2e \neq \lim_{x\to 0^-} f'(x) = 2e$$

A5

in $x=0$ punto angoloso

$f(0)=e$

f cresce in $(-1,0)$; f decresce in $(0,+\infty)$

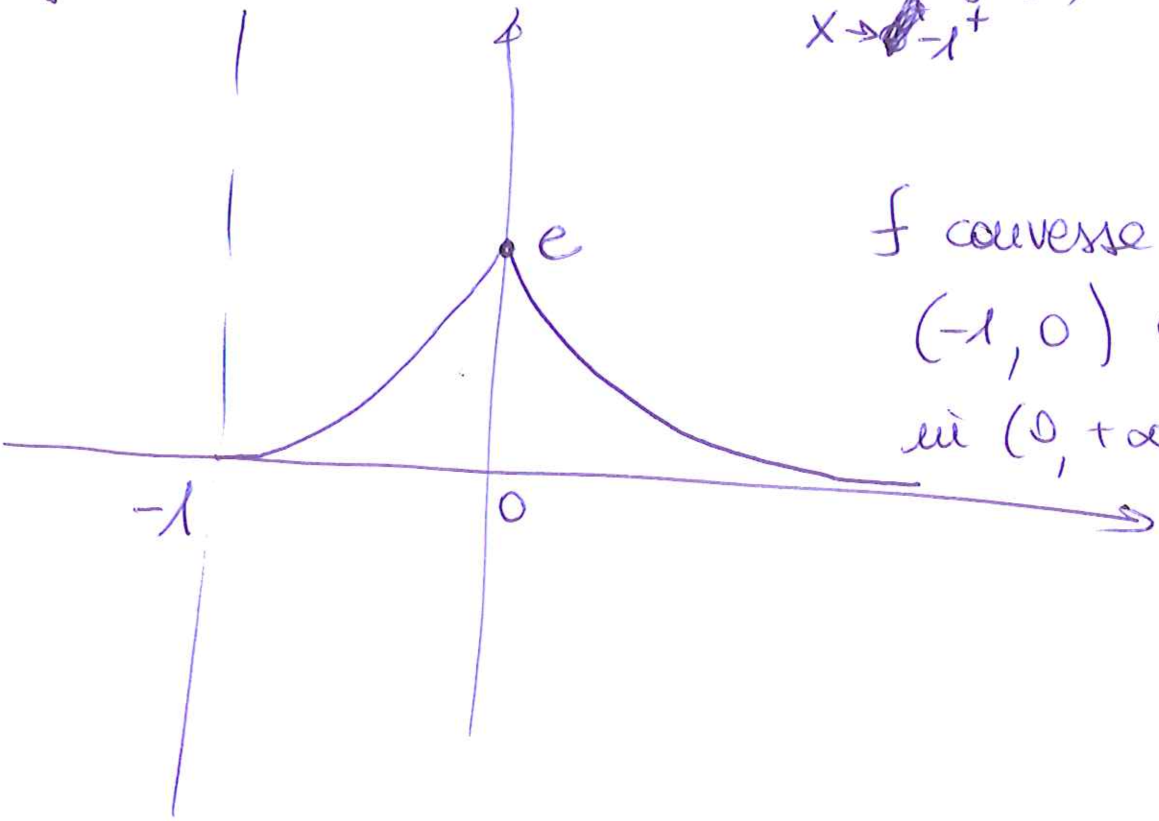
in $x=0$ MAX. ASS.

$\nexists$ MIN. REL. o ASS. ; 0 è $\inf(f)$.

$$f''(x) = \begin{cases} 2e > 0 & \text{se } -1<x<0 \\ \dfrac{6e}{(x+1)^4} > 0 & \text{se } x>0 \end{cases}$$

Grafico:

$$\lim_{x\to -1^+} f'(x) = 0^+$$



f convessa in $(-1,0)$ e in $(0,+\infty)$.

4) $\textcircled{A_6}$

$$0 \leq \frac{3^n - |\sin n|}{5^n} \leq \frac{3^n}{5^n} = \left(\frac{3}{5}\right)^n$$

Poichè $\sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n$ converge (serie geometrica di ragione $\frac{3}{5}$),

allora, per il criterio del confronto, converge la serie di partenza.

Inoltre

$$S < \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n = \frac{1}{1-\frac{3}{5}} = \frac{5}{2}.$$

5) $\cos(\sqrt{x}) \sim 1 - \frac{x}{2}$

$\Rightarrow \log(\cos(\sqrt{x})) \sim -\frac{x}{2}$

$$1 + e^{-\frac{1}{|x|}} \xrightarrow[x \to 0^+]{} 1 + e^{-\infty} = 1$$

$$\sinh\left(\frac{x}{2}\right) - \frac{x}{2} = \cancel{\frac{x}{2}} + \frac{x^3}{48} - \cancel{\frac{x}{2}} + o(x^3) \sim \frac{x^3}{48}$$

$$\Rightarrow f(x) \underset{x \to 0^+}{\sim} \frac{-\frac{x}{2}}{\frac{x^3}{48}} = -\frac{24}{x^2} \xrightarrow[x \to 0^+]{} -\infty$$