COMPITO B

B1

1) $\cosh(\sqrt{x}) \sim 1 + \frac{x}{2}$

$\Rightarrow \log[\cosh(\sqrt{x})] \sim \frac{x}{2}.$

$1 + e^{-\frac{1}{x^2}} \xrightarrow[x \to 0^+]{} 1 + e^{-\infty} = 1.$

$\cos\left(\frac{x}{2}\right) - 1 + \frac{x^2}{8} \underset{x\to 0}{\sim} 1 - \frac{x^2}{8} + \frac{x^4}{16 \cdot 4!} - 1 + \frac{x^2}{8}$

$= \frac{x^4}{384}$

$\Rightarrow f(x) \underset{x\to 0^+}{\sim} \frac{\frac{x}{2}}{\frac{x^4}{384}} = \frac{192}{x^3} \xrightarrow[x\to 0^+]{} +\infty.$

2) $1 - x > 0 \Rightarrow x < 1$

$I_{def} = (-\infty, 1).$ $\quad f(x) > 0 \ \forall x \in I_{def}$

$$f(x) = \begin{cases} e \cdot e^{2[\log(1-x)]} & \text{se } 1-x \geq 1 \\ e \cdot e^{-2[\log(1-x)]} & \text{se } 1-x < 1 \end{cases}$$

$$= \begin{cases} e \cdot (1-x)^2 & \text{se } x \leq 0 \\ \frac{e}{(1-x)^2} & \text{se } 0 < x < 1 \end{cases}$$

$\cancel{f(0)=e}$ B2

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{e}{(1-x)^2} = \frac{e}{0^+} = +\infty$$

AS. VERTICALE SX $x=1$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} e(1-x)^2 = +\infty$$

NO AS. ORIZZONTALE

Poiché per $x\to-\infty$ $f(x)\sim ex^2$, non c'è asintoto obliquo.

$$f'(x) = \begin{cases} -2e(1-x)<0 & \text{se } x<0 \\ \dfrac{2e}{(1-x)^3}>0 & \text{se } 0<x<1 \end{cases}$$

$$\lim_{x\to 0^+} f'(x) = 2e \neq \lim_{x\to 0^-} f'(x) = -2e$$

In $x=0$ PUNTO ANGOLOSO

f decresce in $(-\infty,0)$; cresce in $(0,1)$.

In $x=0$ MIN. ASS: $f(0)=e$.

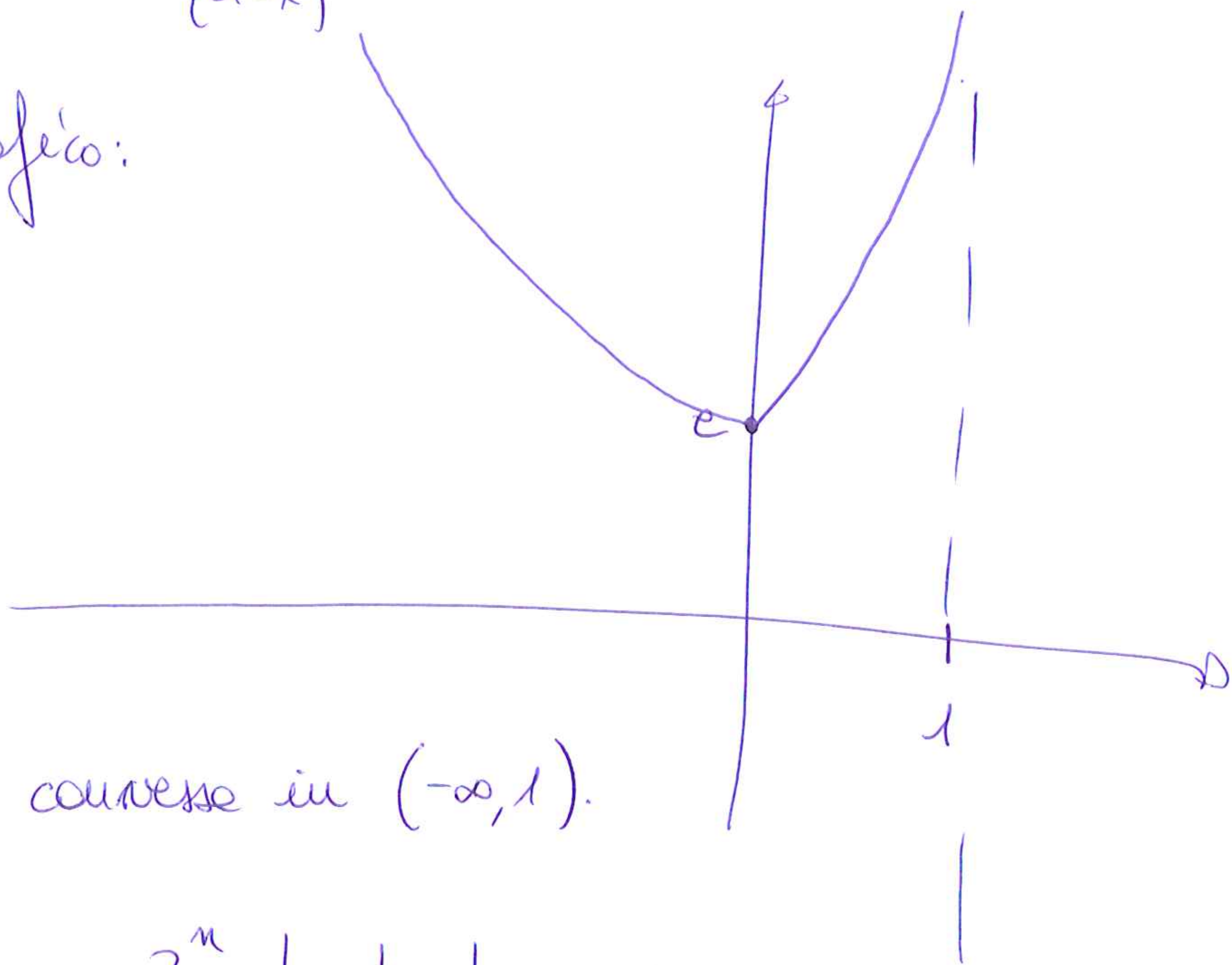
Poiché $\lim_{x\to 1^-} f(x) = \lim_{x\to -\infty} f(x) = +\infty$

f è ILLIMITATA SUP.MENTE.

B3

$$f''(x)=\begin{cases} 2e>0 & \text{se } x<0 \\ \dfrac{6e}{(1-x)^4}>0 & \text{se } 0<x<1 \end{cases}$$

Grafico:



f è convessa in $(-\infty,1)$.

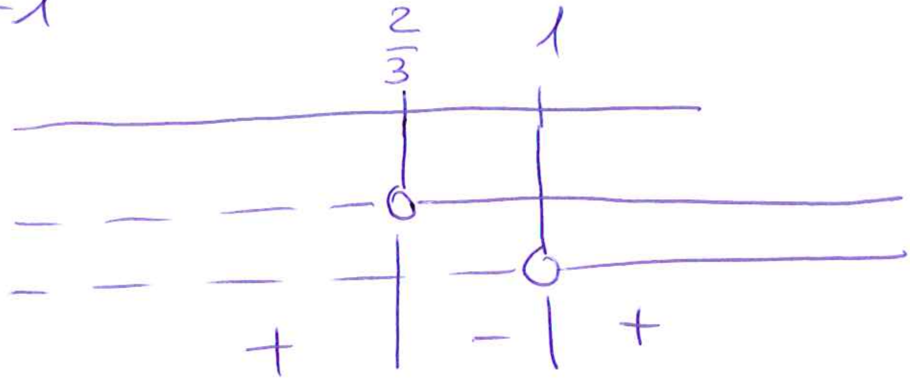
3) $$\frac{2^n-|\operatorname{arctg} n|}{5^n} \leq \frac{2^n}{5^n} = \left(\frac{2}{5}\right)^n$$

Poiché $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n$ converge $\left(\text{serie geometrica di ragione } \dfrac{2}{5}\right)$ per il criterio del confronto converge la serie di partenza.

Inoltre $$S \leq \sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}.$$

4) $\dfrac{-3x+2}{(x^2-4x+3)(x-3)} = \dfrac{-3x+2}{(x-3)^2(x-1)} > 0$ B4

$\Leftrightarrow \dfrac{3x-2}{x-1} < 0$



$\Rightarrow \quad f(x) < 0 \quad \forall x \in [4, +\infty).$

$$\frac{-3x+2}{(x-3)^2(x-1)} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{(x-1)} =$$

$$\frac{A(x-3)(x-1) + B(x-1) + C(x-3)^2}{(x-3)^2(x-1)}$$

$$\Rightarrow \begin{cases} A + C = 0 \\ -4A + B - 6C = -3 \\ 3A - B + 9C = 2 \end{cases} \Rightarrow \begin{cases} C = -A \\ -4A + B + 6A = -3 \\ 3A - B - 9A = 2 \end{cases}$$

$$\Rightarrow \begin{cases} C = -A \\ 2A + B = -3 \\ -6A - B = 2 \end{cases} \Rightarrow \begin{cases} C = -A \\ -4A = -1 \\ B = -3 - 2A \end{cases} \Rightarrow \begin{cases} A = \frac{1}{4} \\ C = -\frac{1}{4} \\ B = -3 - \frac{1}{2} = -\frac{7}{2} \end{cases}$$

$$\int_4^{+\infty} f(x)dx = \int_4^{+\infty} \left[\frac{1}{4} \cdot \frac{1}{(x-3)} - \frac{7}{2(x-3)^2} - \frac{1}{4(x-1)} \right] dx$$

B5

$$= \left[\frac{1}{4} \log\left(\frac{x-3}{x-1}\right) + \frac{7}{2(x-3)} \right]_4^{+\infty}$$

$$= \lim_{x\to\infty} \left[\frac{1}{4} \log\left(\frac{x-3}{x-1}\right) + \frac{7}{2(x-3)} \right] - \frac{1}{4} \log\left(\frac{1}{3}\right) - \frac{7}{2}$$

$$= -\frac{7}{2} + \frac{1}{4} \log(3).$$

La funzione è integrabile in quanto, per $x \to +\infty$,

$$f(x) \sim \frac{-3x}{x^3} = -\frac{3}{x^2}$$

che è integrabile in $[4, +\infty)$.

5) $z \neq -i$

$$z + \frac{3}{2} - i + z(z+i) = 0$$

$$z^2 + (1+i)z + \frac{3}{2} - i = 0$$

$$z_{1,2} = \frac{-(1+i) \pm \sqrt{(1+i)^2 - 6 + 4i}}{2} = \frac{-(1+i) \pm \sqrt{6i-6}}{2}$$

Poichè $|6i-6| = 6\sqrt{2}$ (B6)

$$\Rightarrow 6i-6 = 6\sqrt{2}\left[-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$$

$$= \cancel{6\sqrt{2}}\ \sqrt{72}\left[\cos\left(\frac{3}{4}\pi\right) + i\,\mathrm{sen}\left(\frac{3}{4}\pi\right)\right]$$

$$\Rightarrow \sqrt{6i-6} = \pm\sqrt{72}\left[\cos\left(\frac{3}{8}\pi\right) + i\,\mathrm{sen}\left(\frac{3}{8}\pi\right)\right]$$

$$\Rightarrow z_{1,2} = \frac{-(1+i) \pm \sqrt{72}\left[\cos\left(\frac{3}{8}\pi\right) + i\,\mathrm{sen}\left(\frac{3}{8}\pi\right)\right]}{2}$$

Usando le formule di bisezione

$$\cancel{\cos\left(\frac{3}{8}\pi\right)} \quad \cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\mathrm{sen}\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

e tenendo conto che $\frac{3}{8}\pi$ si trova nel primo quadrante, si ha

$$\cos\left(\frac{3}{8}\pi\right) = \sqrt{\frac{1+\cos\left(\frac{3}{4}\pi\right)}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$$

$$\mathrm{sen}\left(\frac{3}{8}\pi\right) = \sqrt{\frac{1-\cos\left(\frac{3}{4}\pi\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$$

da cui

$$z_{1,2} = \frac{-(1+i) \pm \frac{\sqrt{72}}{2}\left[\sqrt{2-\sqrt{2}} + i\sqrt{2+\sqrt{2}}\right]}{2}$$

$$= \frac{-(1+i) \pm 3\sqrt{2}\left[\sqrt{2-\sqrt{2}} + i\sqrt{2+\sqrt{2}}\right]}{2}$$

B_7