SVOLGIMENTI di ANALISI 1 del 7/9/2021

1) $I_{def} = \{x \neq 1\} = \mathbb{R} - \{1\}$.

Intersezioni con assi: $f(0) = -1$ asse y

$f(x) = 0 \iff x = -1$ asse x.

Segno: $f(x) > 0 \iff x - 1 > 0$

$\iff x > 1$.

Limiti: $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2}{x^3} = 0^{\pm}$

AS. ORIZZONTALE $y = 0$ a $\pm\infty$

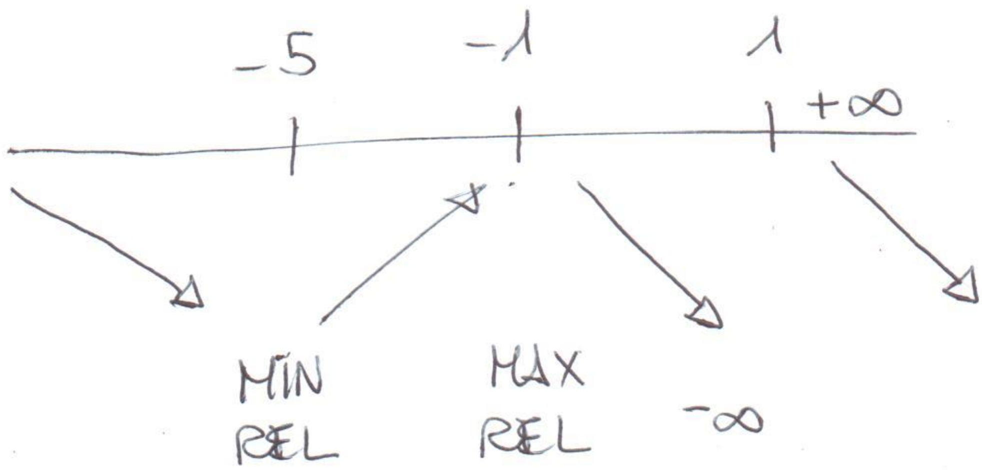
$\lim_{x \to 1^{\pm}} f(x) = \frac{4}{(0^{\pm})^3} = \pm\infty$ AS. VERTICALE $x = 1$ da DX e SX.

$$f'(x) = \frac{2(x+1)(x-1)^{\cancel{3}} - (x+1)^2 3\cancel{(x-1)^2}}{(x-1)^{\cancel{6}4}}$$

$$= \frac{(x+1)}{(x-1)^4}\left[2x-2-3x-3\right] = \frac{-(x+1)(x+5)}{(x-1)^4}$$

$f'(x) > 0 \iff -5 < x < -1$

②



$\nexists$ MAX. o MIN. ASS. perchè $\lim_{x \to 1^\pm} f(x) = \pm\infty$

$$f'(x) = -\frac{(x^2+6x+5)}{(x-1)^4}$$

$$\Rightarrow f''(x) = -\left[\frac{(2x+6)(x-1)^{\cancel{4}} - (x^2+6x+5)\,4\,\cancel{(x-1)}^{3}}{(x-1)^{\cancel{8}\,5}}\right]$$

$$= -\frac{2}{(x-1)^5}\left[(x+3)(x-1) - 2(x^2+6x+5)\right]$$

$$= -\frac{2}{(x-1)^5}\left(x^2+2x-3-2x^2-12x-10\right)$$

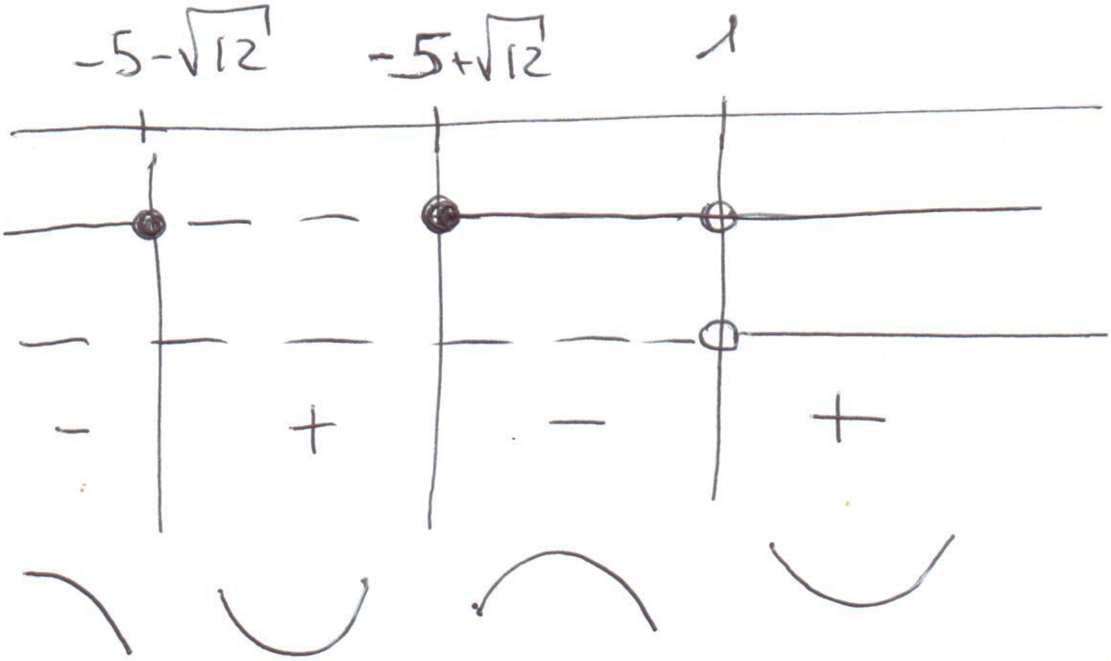
$$= +\frac{2}{(x-1)^5}\left(x^2+10x+13\right)$$

$\cancel{x^2-10x+1}$

Studiamo $x^2+10x+13=0$

$$x_{1,2} = -5 \pm \sqrt{25-13} = -5 \pm \sqrt{12}$$

N.B.: $-2 < -5+\sqrt{12} < -1$

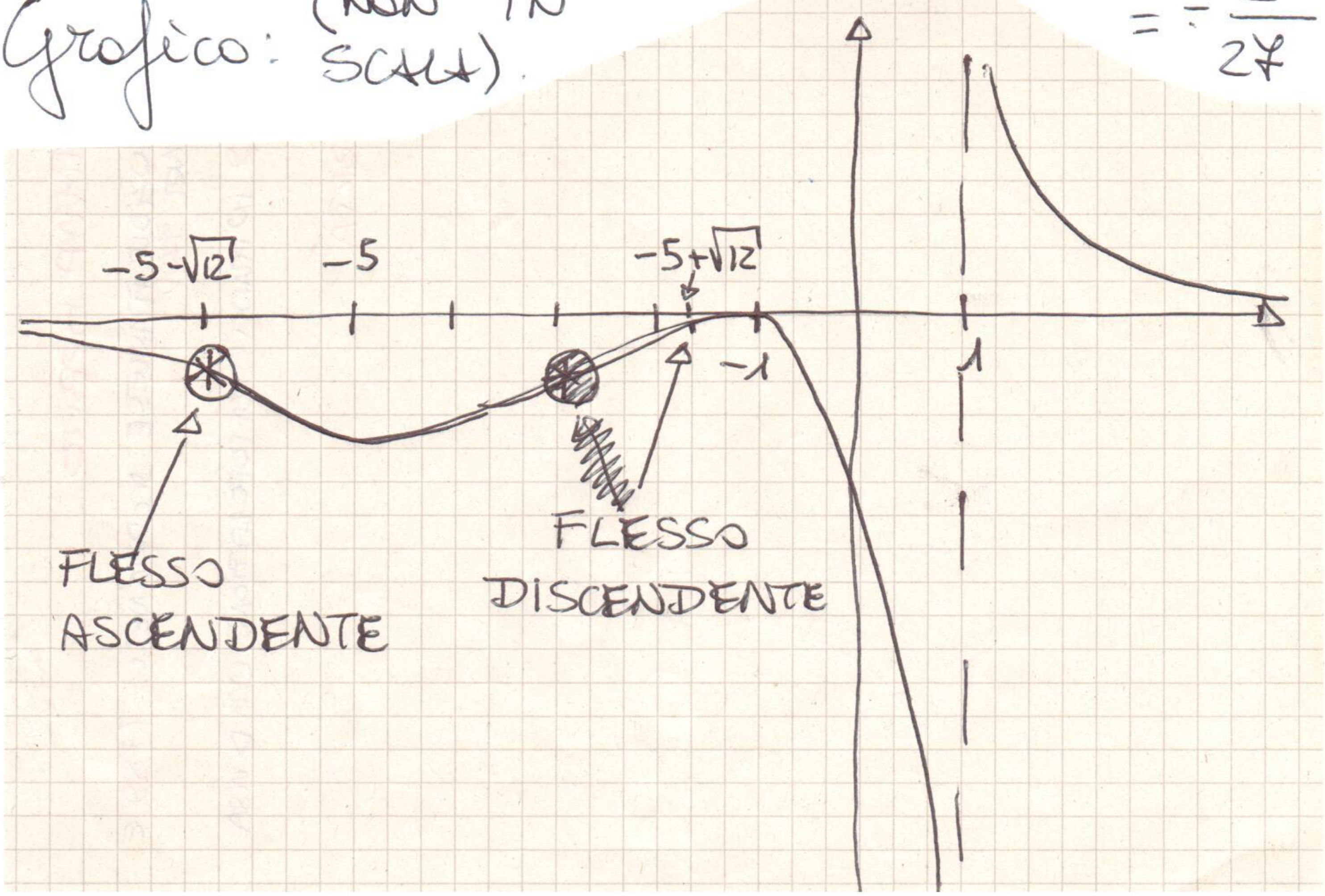


$f(-1)=0$

$f(-5)=$

$$\frac{(-4)^2}{(-6)^3} = \frac{2^{\cancel{4}}}{-\cancel{2^3}\cdot 3^3} = -\frac{2}{27}$$

Grafico: (NON IN SCALA)



④

2) $$\lim_{x\to 0} \frac{\left[\cancel{x}-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\cancel{-x}\right]^2-\frac{1}{4}x^4}{x^5}$$

$$=\lim_{x\to 0} \frac{x^4\left[-\frac{1}{2}+\frac{x}{3}+o(x)\right]^2-\frac{1}{4}x^4}{x^5}$$

$$=\lim_{x\to 0} \frac{\left(\frac{1}{4}+\frac{x^2}{9}+o(x)-\frac{x}{3}\right)-\frac{1}{4}}{x}$$

doppio prodotto di $-\frac{1}{2}$ e $o(x)$

doppio prodotto di $-\frac{1}{2}$ e $\frac{x}{3}$

$$=\lim_{x\to 0} \frac{-\frac{x}{3}+o(x)}{x} = -\frac{1}{3}.$$

3) $$(2+i)(i+1)\pi i = (2+3i-1)\pi i$$

$$= \pi(-3+i)$$

$$\Rightarrow e^{(2+i)(i+1)\pi i} = e^{-3\pi}\cdot e^{+i\pi}$$

Ma $e^{+i\pi}=-1$.

$$\Rightarrow \; = -e^{-3\pi}$$

$$\Rightarrow \left| e^{(2+i)(i+1)\pi i} \right| = e^{-3\pi}$$ 5

$$Re = -e^{-3\pi} \qquad Im = 0.$$

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4) $a_0 = 0$

Inoltre $\forall n \geq 1 \quad 0 < \frac{\pi}{2^n} \leq \frac{\pi}{2}$

$$\Rightarrow \sin\left(\frac{\pi}{2^n}\right) > 0$$

$$\Rightarrow a_n \geq 0 \quad \forall n \in \mathbb{N}$$

$$a_n > 0 \quad \forall n \geq 1.$$

Inoltre, $\sin\left(\frac{\pi}{2^n}\right) \underset{n\to\infty}{\sim} \frac{\pi}{2^n}$

$$\Rightarrow a_n \underset{n\to\infty}{\sim} \frac{\pi n^2}{2^n} \xrightarrow[n\to\infty]{} 0$$ per gli ordini di infinito

Serie: criterio del confronto asintotico:

$$\sum a_n \approx \sum \frac{\pi n^2}{2^n}$$ criterio della radice

$$\sqrt[n]{a_n} = \frac{\sqrt[n]{\pi n^2}}{2} \xrightarrow[n\to\infty]{} \frac{1}{2} < 1$$

$\Rightarrow \sum a_n$ converge.

5) Sostituzione: $t=\sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}} dx$ $(x>0)$

$\Rightarrow dx = 2t\,dt$

$$\Rightarrow 2\int \frac{(1-t)}{(1+t)} t\,dt \Big|_{t=\sqrt{x}}$$

$$= -2\int \left[t-2 + \frac{2}{t+1} \right] dt \Big|_{t=\sqrt{x}}$$

$$= -2 \left[\frac{t^2}{2} - 2t + 2\ln|t+1| \right] \Big|_{t=\sqrt{x}} + c$$

$$= -x + 4\sqrt{x} - 4\ln|\sqrt{x}+1| + c$$

$$= -x + 4\sqrt{x} - 4\ln(\sqrt{x}+1) + c$$