SVOLGIMENTI PROVA SCRITTA DI ANALISI 1 del 9/6/2022

(1)

1) Insieme di definizione:

$$\begin{cases} x^2+2x \neq 0 \\ x+2 \geq 0 \end{cases} \Rightarrow \begin{cases} x \neq 0;\ x \neq -2 \\ x \geq -2 \end{cases}$$

$$\Rightarrow (-\infty,-2) \cup (-2,0) \cup (0,+\infty)$$

Poiché $x_0 = 1 \Rightarrow I = (0,+\infty)$.

$$\frac{-x+1}{x^2+2x} \in C^\infty(0,+\infty);\ \sqrt{x+2} \in C^\infty(0,+\infty)$$

$\Rightarrow \exists !$ sol. GLOBALE in $(0,+\infty)$.

$$y(x) = e^{\int_1^x \frac{t+1}{t^2+2t}dt}\left[\int_1^x e^{-\int_1^t \frac{s+1}{s^2+2s}ds}\sqrt{t+2}\,dt\right]$$

$$= e^{\frac{1}{2}\ln(|t^2+2t|)\big|_1^x}\left[\int_1^x e^{-\frac{1}{2}\ln(|s^2+2s|)\big|_1^t}\sqrt{t+2}\,dt\right]$$

Ma in $(0,+\infty)$ $t^2+2t>0$

$$= e^{\frac{1}{2}\ln\left(\frac{x^2+2x}{3}\right)}\left[\int_1^x e^{-\frac{1}{2}\ln\left(\frac{t^2+2t}{3}\right)}\sqrt{t+2}\,dt\right]$$

$$= \sqrt{\frac{x^2+2x}{\cancel{3}}}\left[\int_1^x \sqrt{\frac{\cancel{3}}{t^2+2t}}\sqrt{t+2}\,dt\right]$$

$$= \sqrt{x^2+2x}\left[\int_1^x \frac{1}{\sqrt{t}\,\cancel{\sqrt{t+2}}}\cancel{\sqrt{t+2}}\,dt\right]$$

②

$$\Rightarrow y(x) = \sqrt{x^2+2x}\left[2\sqrt{t}\right]_1^x$$

$$= 2\sqrt{x^2+2x}\left(\sqrt{x}-1\right).$$

2) $\sqrt{x} = t \qquad dt = \frac{1}{2\sqrt{x}}\,dx \Rightarrow dx = 2t\,dt$

$$t(0)=0 \qquad t(1)=1$$

$$\Rightarrow \int_0^1 \frac{2t^2}{(t^2+1)(t+1)}\,dt$$

$$\frac{2t^2}{(t^2+1)(t+1)} = \frac{At+B}{t^2+1} + \frac{C}{t+1} = \frac{(At+B)(t+1)+C(t^2+1)}{(t^2+1)(t+1)}$$

$$\Rightarrow \begin{cases} A+C=2 \\ A+B=0 \\ B+C=0 \end{cases} \Rightarrow \begin{cases} A=-B \\ C=-B \\ B=-1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=1 \end{cases}$$

$$\Rightarrow \int_0^1 \left[\frac{t-1}{t^2+1} + \frac{1}{t+1}\right]dt$$

$$= \int_0^1 \left[\frac{1}{2}\cdot\frac{2t}{t^2+1} - \frac{1}{t^2+1} + \frac{1}{t+1}\right]dt$$

$$= \left[\frac{1}{2}\ln(t^2+1) - \operatorname{arctg} t + \ln(|t+1|)\right]_0^1$$

$$= \frac{1}{2}\ln 2 - \operatorname{arctg} 1 + \ln 2 = \frac{3}{2}\left[\ln 2\right] - \frac{\pi}{4}$$

$$\cancel{= \frac{1}{2}\ln 4 - \frac{\pi}{4}}$$

3

3)

$$\left|e^{i(z+i)}\right| = \left|e^{i(a+ib+i)}\right| = \left|e^{ia}\cdot e^{-(b+1)}\right|$$

$$= e^{-(b+1)} < 1 \qquad \left(\begin{matrix}\text{disequazione}\\ \text{in } \mathbb{R}\end{matrix}\right)$$

$$\Rightarrow \; -b-1<0 \;\Rightarrow \begin{cases} b>-1 \\ a\in\mathbb{R}\end{cases}$$

4)

$$a_n = \frac{\left(-\cancel{\frac{1}{n^3}} - \cancel{\frac{1}{2n^6}} - \frac{1}{3n^9}\right) + \left(\cancel{\frac{1}{n^3}} + \cancel{\frac{1}{2n^6}} + \frac{1}{6n^9}\right) + o\left(\frac{1}{n^9}\right)}{\left(\cancel{\frac{1}{n^2}} - \frac{1}{3n^6}\right) - \left(\cancel{\frac{1}{n^2}} + \frac{1}{3n^6}\right) + o\left(\frac{1}{n^6}\right)}$$

$$= \frac{\left(-\frac{1}{3}+\frac{1}{6}\right)\frac{1}{n^9} + o\left(\frac{1}{n^9}\right)}{\left(-\frac{1}{3}-\frac{1}{3}\right)\frac{1}{n^6} + o\left(\frac{1}{n^6}\right)} \sim \frac{-\frac{1}{6}\frac{1}{n^9}}{-\frac{2}{3}\frac{1}{n^6}}$$

$$= \frac{1}{4n^3}$$

$\sum \frac{1}{4n^3}$ converge $\Rightarrow \sum a_n$ converge.

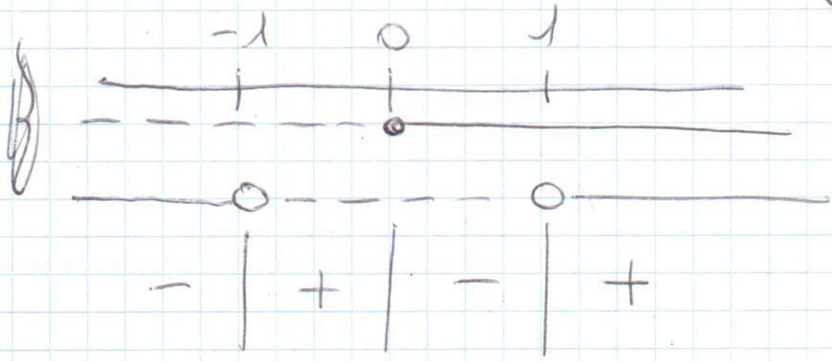
5) $I_{def} = \{x \neq \pm 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$.

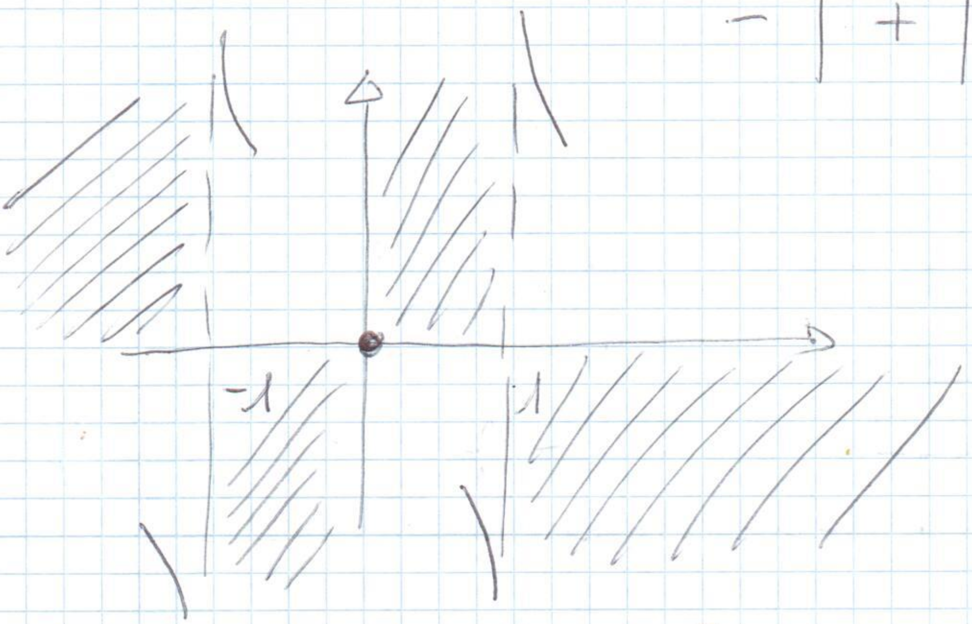
f DISPARI

$f(x) = 0 \iff x = 0$

4

$f(x) > 0 \iff$





$$\lim_{x \to 1^\pm} f(x) = \lim_{x \to 1^\pm} \frac{x^3}{(x+1)(x-1)} = \frac{1}{2 \cdot 0^\pm} = \pm\infty$$

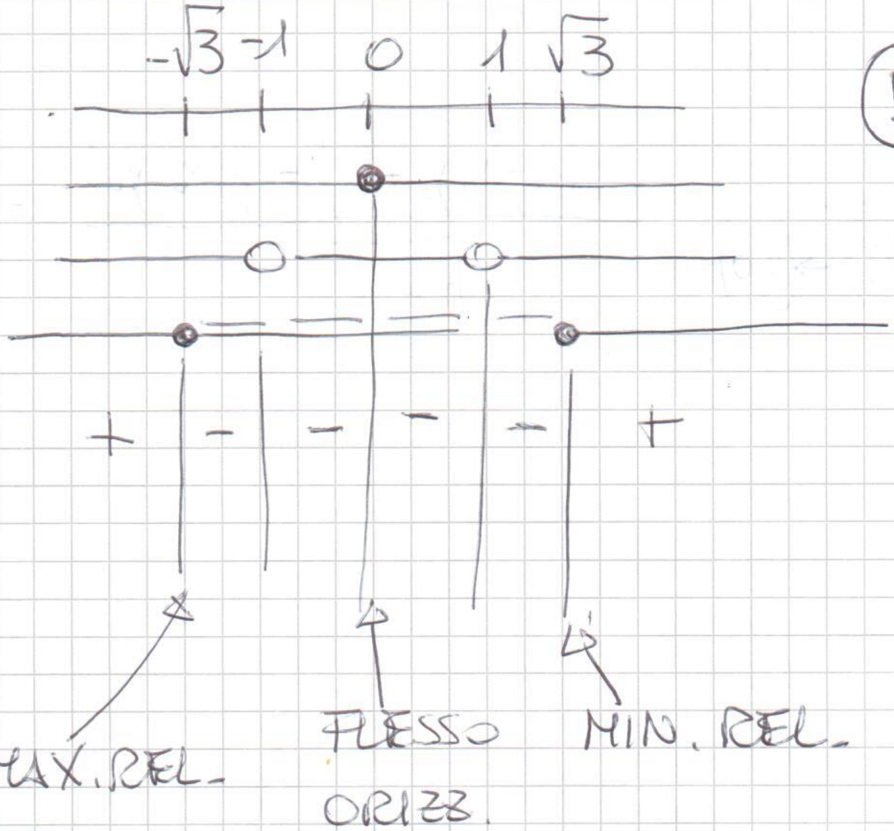
$\Rightarrow$ per disparità $\lim_{x \to -1^\pm} = \pm\infty$ AS. VERT. $x = \pm 1$.

$$f(x) = \frac{x^3 - x + x}{x^2 - 1} = x + \frac{x}{x^2 - 1} = x + o(1) \quad \text{per } x \to \pm\infty$$

$\Rightarrow$ AS. OBLIQUO $y = x$ per $x \to \pm\infty$

$$f'(x)=\frac{3x^2(x^2-1)-x^3\,2x}{(x^2-1)^2}=\frac{x^4-3x^2}{(x^2-1)^2}=\frac{x^2(x^2-3)}{(x^2-1)^2}$$

$f'(x)\geq 0$



$$f(\sqrt{3})=\frac{3\sqrt{3}}{2}=\frac{3}{2}\sqrt{3}$$

$$f(-\sqrt{3})=-\frac{3}{2}\sqrt{3}$$

Poiché $f(x)\xrightarrow[x\to\pm\infty]{}\pm\infty$

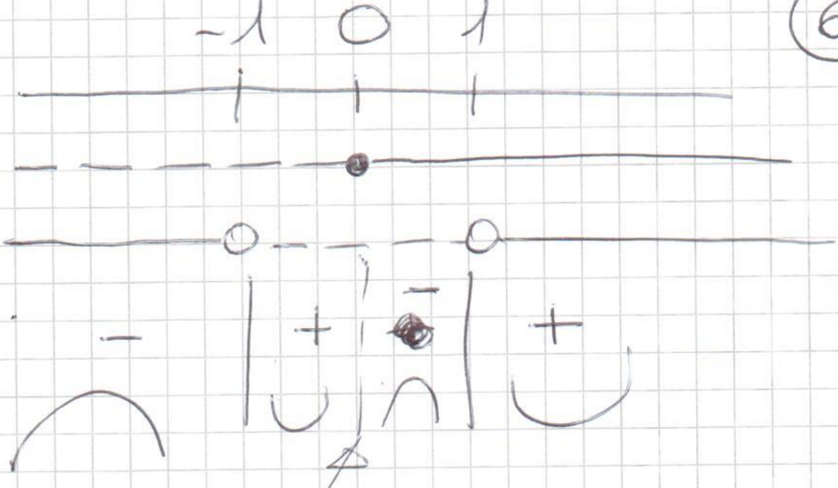
$\nexists$ MAX. MIN. ASSOLUTI

$$f''(x)=\frac{(4x^3-6x)(x^2-1)^{\cancel{2}}-(x^4-3x^2)2\cancel{(x^2-1)}2x}{(x^2-1)^{\cancel{4}3}}$$

$$=\frac{\cancel{4x^5}-4x^3-6x^3+6x-\cancel{4x^5}+12x^3}{(x^2-1)^3}$$

$$=\frac{2x^3+6x}{(x^2-1)^3}=\frac{2x(x^2+3)}{(x^2-1)^3}$$

$f(x) \geq 0$



flesso (orizzontale)
GIÀ CONSIDERATO

GRAFICO: