COMPITO B

1)
$$a_n = n^3\left[\ln\left(1+\frac{1}{n^2}\right) - \sin\left(\frac{1}{n^2}\right)\right] =$$

$$= n^3\left[\cancel{\frac{1}{n^2}} - \frac{1}{2n^4} + \frac{1}{3n^6} + o\left(\frac{1}{n^6}\right) - \cancel{\frac{1}{n^2}} + \frac{1}{6n^6} + o\left(\frac{1}{n^6}\right)\right]$$

$$= n^3\left[-\frac{1}{2n^4} + o\left(\frac{1}{n^4}\right)\right] \underset{n\to\infty}{\sim} -\frac{1}{2n}$$

Poichè la serie $\sum -\frac{1}{2n}$ diverge, allora la serie diverge (negativamente).

2) $\underline{I_{def}}$: $e^{3x}-1>0 \Rightarrow e^{3x}>1$

$\Rightarrow x>0$

$\underline{\text{Segno}}$: $f(x)=0 \iff e^{3x}-1=1$

$\iff e^{3x}=2 \iff 3x=\ln 2 \iff x=\frac{\ln 2}{3}$

$f(x)>0 \iff x>\frac{\ln 2}{3}$.

ASINTOTI: $\lim_{x \to 0^+} f(x) = \ln(0^+) = -\infty$ (B2)

AS. VERTICALE DESTRO
$x = 0$.

Per $x \to +\infty$: $\ln(e^{3x} - 1) = \ln(e^{3x}) + \ln(1 - e^{-3x})$

$= 3x + o(1)$.

$\Rightarrow$ $y = 3x$ ASINTOTO OBLIQUO
per $x \to +\infty$.

MONOTONIA: $f'(x) = \dfrac{3e^{3x}}{e^{3x} - 1} > 0 \quad \forall x > 0$

f sempre crescente

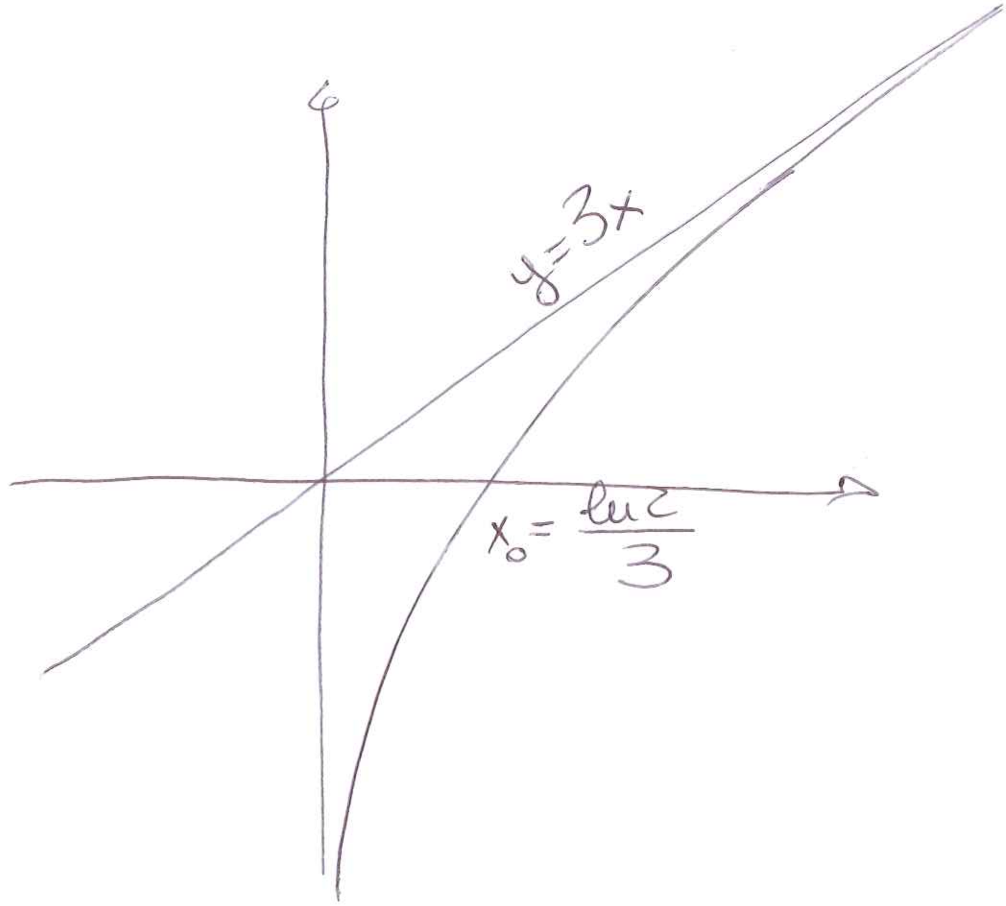
CONVESSITÀ $f''(x) = 3\left[\dfrac{3e^{3x}(e^{3x} - 1) - 3e^{6x}}{(e^{3x} - 1)^2}\right]$

$= \dfrac{-9e^{3x}}{(e^{3x} - 1)^2} > 0 \quad \forall x > 0$

f sempre CONCAVA.

B_3

Grafico:



3) OMOGENEA ASSOCIATA:

$$\alpha^2+1=0$$

$$\Rightarrow y_0(x) = C_1 \operatorname{sen} x + C_2 \cos x$$

Eq.ne COMPLETA:

$$y_p(x) = Ax^3+Bx^2+Cx+D$$

$$y_p'(x) = 3Ax^2+2Bx+C$$

$$y_p''(x) = 6Ax+2B$$

$$6Ax+2B+Ax^3+Bx^2+Cx+D=-x^3$$

B_4

$$\begin{cases} A=-1 \\ B=0 \\ 6A+C=0 \\ 2B+D=0 \end{cases} \Rightarrow \begin{cases} A=-1 \\ B=0 \\ C=-6A=6 \\ D=0 \end{cases}$$

$$\Rightarrow y(x) = C_1 \sin x + C_2 \cos x - x^3 + 6x$$

$$y(0) = C_2 = 0$$

$$y(\pi) = -\pi^3 + 6\pi = 0 \quad \leftarrow \text{IMPOSSIBILE.}$$

Quindi non esistono soluzioni del problema ai bordi.

4) a) $f \in C^0\left(0, \frac{\pi}{4}\right]$.

$f(x) \underset{x \to 0}{\sim} \frac{1}{x}$ NON INTEGRABILE

b) $\displaystyle\int \frac{1-tg^2 x}{tg x}\,dx = \int \frac{1}{tg x}\,dx - \int tg x\,dx$

$= \ln|\sin x| + \ln|\cos x| + C$

c) Per il punto a) e poiché in $\left(0, \frac{\pi}{4}\right]$ B_S

$$f(x) = \frac{1-\operatorname{tg}^2 x}{\operatorname{tg} x} \geq 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(x)\,dx = +\infty$$

Direttamente, si ha

$$\int_0^{\frac{\pi}{4}} f(x)\,dx = \Big[\ln|\operatorname{sen} x| + \ln|\cos x|\Big]_0^{\frac{\pi}{4}}$$

$$= 2\ln\left(\frac{\sqrt{2}}{2}\right) - \lim_{x\to 0^+} \ln|\operatorname{sen} x| =$$

$$= -\ln 2 - \ln(0^+) = +\infty.$$

5) $\operatorname{Re}(z) \neq 0 \Rightarrow x \neq 0.$

$$i(x-iy) + \operatorname{Im}\left(\frac{1}{3} + \frac{i}{3}\right) + y = 0$$

$$ix + y + \frac{1}{3} + y = 0$$

$$\begin{cases} x = 0 \longleftarrow \text{soluzione scartata} \\ 2y + \frac{1}{3} = 0 \end{cases}$$

$\Rightarrow \nexists$ soluzioni.