SVOLGIMENTO PROVA SCRITTA ANALISI 1
del 13/4/2012

1) $|z^2| = |z \cdot z| = |z| \cdot |z| \implies$

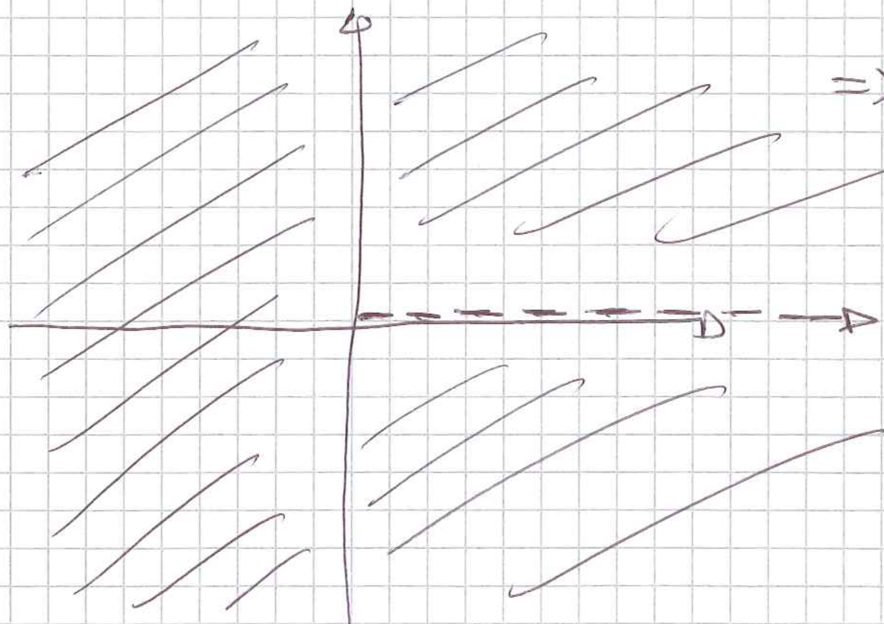
$$|z|\left(|z| - \mathrm{Re}(z)\right) \geq 0$$

Poiché $|z| \geq 0 \quad \forall z \in \mathbb{C}$
e $|z| = 0 \iff z = 0$ $\implies \begin{cases} z \neq 0 \\ |z| - \mathrm{Re}(z) \geq 0 \end{cases}$

$$\begin{cases} z \neq 0 \\ \sqrt{x^2+y^2} - x \geq 0 \end{cases} \qquad \begin{cases} z \neq 0 \\ \sqrt{x^2+y^2} \geq x \end{cases}$$

$$\begin{cases} z \neq 0 \\ x < 0 \\ \forall (x,y) \in \mathbb{R}^2 \end{cases} \cup \begin{cases} z \neq 0 \\ x \geq 0 \\ \cancel{x^2} + y^2 > \cancel{x^2} \end{cases}$$

$$\begin{cases} y \in \mathbb{R} \\ x < 0 \end{cases} \cup \begin{cases} z \neq 0 \\ x \geq 0 \\ y^2 > 0 \end{cases} \implies \begin{cases} y \in \mathbb{R} \\ x < 0 \end{cases} \cup \begin{cases} y \neq 0 \\ x \geq 0 \end{cases}$$



$$\implies \mathbb{C} - \left\{ (x,y) \;\middle|\; \begin{matrix} x \geq 0 \\ y = 0 \end{matrix} \right\}$$

2) $\log(1+x^9) \underset{x\to 0}{\sim} x^9$

$\sin(t) \underset{t\to 0}{\sim} t \implies \sin\left[\log(1+x^9)\right] \underset{x\to 0}{\sim} \sin(x^9)$

$\underset{x\to 0}{\sim} x^9$

$3\left[x^3-\sin(x^3)\right] = 3\left[x^3-x^3+\frac{x^9}{3!}+o(x^9)\right]$

$\underset{x\to 0}{\sim} \frac{x^9}{2}$

$\implies \lim_{x\to 0} \frac{\sin\left[\log(1+x^9)\right]}{3x^3-3\sin(x^3)} = \lim_{x\to 0} \frac{x^9}{\left(\frac{x^9}{2}\right)} = 2.$

3) $I_{def} = \left\{x\in\mathbb{R} \mid x>1\right\}$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{(x-1)\log(x-1)+1}{x-1} = \lim_{t\to 0^+} \frac{\overbrace{t\log t}^{\to 0}+1}{t} = \frac{1}{0^+}$

$(t=x-1)$ $\qquad = +\infty$

AS. VERTICALE DX : $x=1$.

$\lim_{x\to +\infty} f(x) = +\infty + 0 = +\infty$ $\nexists$ AS. ORIZZ. DX

$\lim_{x\to +\infty} \frac{f(x)}{x} = \lim_{x\to +\infty}\left[\frac{\log(x-1)}{x}+\frac{1}{(x-1)x}\right] = 0$ $\nexists$ AS. OBL.

$f'(x) = \frac{1}{x-1} - \frac{1}{(x-1)^2} = \frac{x-2}{(x-1)^2} > 0 \iff x>2$

f decresce in $(0, 2)$, cresce in $(2, +\infty)$.

$x_m = 2$ punto di MIN. ASS.

$$f(2) = 1 > 0 \quad \Rightarrow \quad f(x) > 0 \quad \forall x \in I_{def}.$$

$$f''(x) = \frac{(x-1)^{\cancel{2}} - 2\cancel{(x-1)}(x-2)}{(x-1)^{\cancel{4}\,3}} = \frac{-x+3}{(x-1)^3} > 0$$

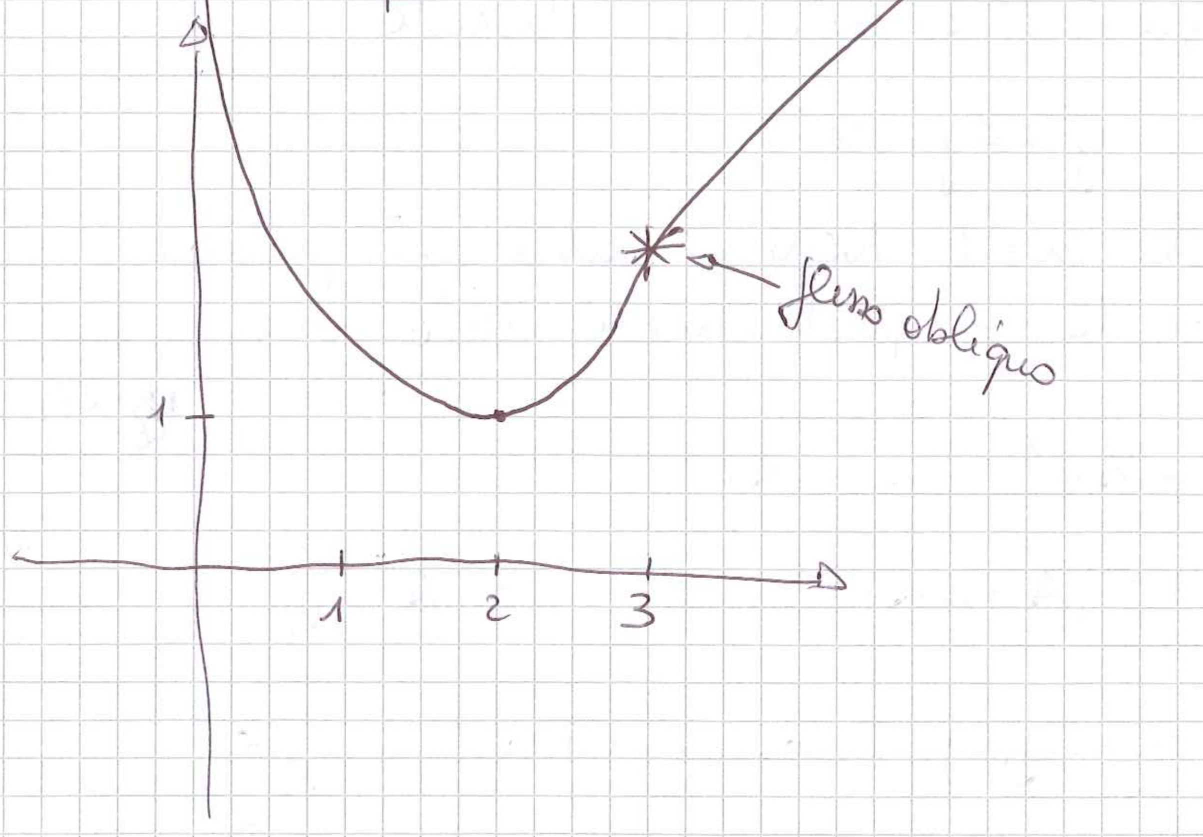
$$\Leftrightarrow \quad x < 3$$

f è convessa in $(0, 3)$ e concava in $(3, +\infty)$

$x_f = 3$ è punto di flesso discendente.

$$f(3) = \log 2 + \frac{1}{2}$$

$$f'(3) = \frac{1}{4}$$



4) Poiché $x^{\alpha} e^{-x}$ è integrabile in $[0, +\infty)$ $\forall \alpha \in \mathbb{R}^+$, la funzione è integrabile. ④

Inoltre, $\int_0^{+\infty} x^n e^{-x} dx = n!$

$$\Rightarrow \int_0^{+\infty} (3x^2 + x - 2) e^{-x} dx = 3 \cdot 2 + 1 - 2 = 5.$$

5) Omogenea associata: $\lambda^2 + 4 = 0 \Rightarrow \lambda_{1,2} = \pm 2i$

$$\Rightarrow y_0(x) = C_1 \cos(2x) + C_2 \operatorname{sen}(2x)$$

Poiché $2(\cosh x + \sinh x) = 2\left[\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}\right]$

$$= 2e^x$$

l'equazione non omogenea diventa

$$y'' + 4y = 2e^x$$

Poiché $\alpha = 1$ NON è radice del polinomio caratteristico, la sol. particolare è della forma

$$w(x) = Ae^x$$

$w' = Ae^x$; $w'' = Ae^x$

$$\Rightarrow (A + 4A) e^x = 2e^x \Rightarrow A = \frac{2}{5}$$

$$\Rightarrow y(x) = C_1 \cos(2x) + C_2 \operatorname{sen}(2x) + \frac{2}{5} e^x$$

$$y(0) = C_1 + \frac{2}{5} = 1 \Rightarrow C_1 = \frac{3}{5}$$

$$y'(x) = -2C_1 \sin(2x) + 2C_2 \cos(2x) + \frac{2}{5} e^x$$

$$y'(0) = 2C_2 + \frac{2}{5} = 1 \qquad \Rightarrow C_2 = \frac{3}{10}$$

$$\Rightarrow \quad y(x) = \frac{3}{5} \cos(2x) + \frac{3}{10} \sin(2x) + \frac{2}{5} e^x$$