SVOLGIMENTO PROVA SCRITTA di ANALISI 1 del 14/1/2021

1) DOMINIO:

$$\begin{cases} x \in [-1,1] & \leftarrow \text{dominio dell'arcoseno} \\ x \in [-1,1] & \leftarrow \text{dominio della radice} \end{cases}$$

$\Rightarrow D = [-1,1]$. $f \in C^0(D)$.

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \frac{\overset{\geq 0}{1-x}}{\underbrace{\sqrt{1-x^2}}_{\cancel{-1<x<x}}} = \sqrt{\frac{(1-x)^2}{(1-x)(1+x)}}$$

$$= \sqrt{\frac{1-x}{1+x}} \geq 0 \quad \cancel{\forall x} \quad -1 < x < 1$$

f crescente in tutto il dominio

$$\lim_{x \to 1^-} f'(x) = 0 \quad ; \quad \lim_{x \to -1^+} f'(x) = \sqrt{\frac{2}{0^+}} = +\infty$$

MINIMO ASSOLUTO in $x = -1$: ~~f(-1)~~

$$f(-1) = \arcsin(-1) = -\frac{\pi}{2}$$

MASSIMO ASSOLUTO in $x = 1$:

$$f(1) = \arcsin(1) = \frac{\pi}{2}$$

$f(0) = 1$

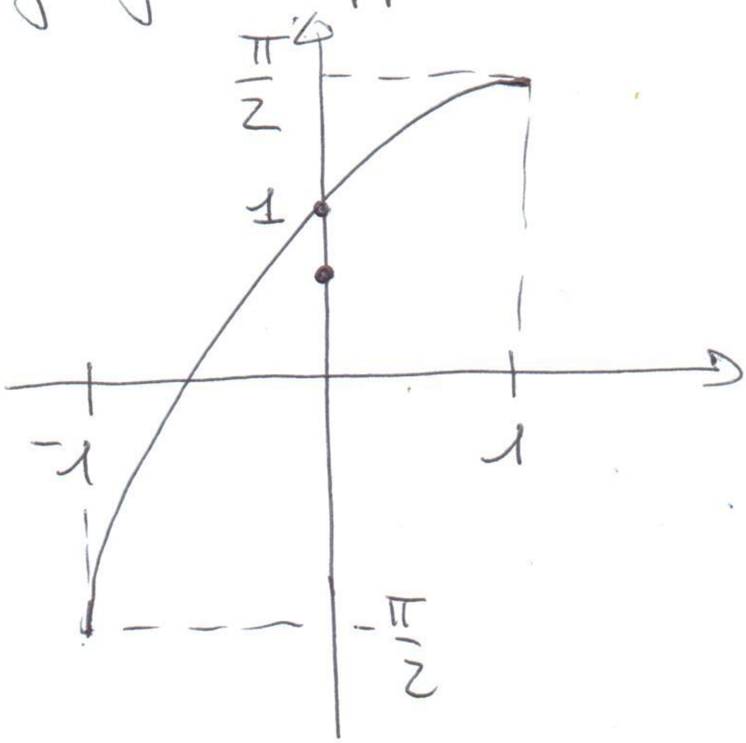
$$f''(x) = \frac{1}{2\sqrt{\frac{1-x}{1+x}}}\left(\frac{1-x}{1+x}\right)' = 2\sqrt{\frac{1+x}{1-x}}\left(\frac{-(1+x)-(1-x)}{(1+x)^2}\right)$$

$$= \frac{1}{2}\sqrt{\frac{(1+x)}{(1-x)(1+x)^4}}\,(-2) = \frac{-1}{\sqrt{(1-x)(1+x)^3}} < 0$$

per $-1 < x < 1$

f sempre concava in D.

Un grafico approssimativo (non richiesto) è



(3)

2)
$$\lim_{x\to 0} \frac{\sinh(2x^2)-\cosh(x^3)-\mathrm{arctg}(2x^2)+1}{\sin^6 x}$$

$$= \lim_{x\to 0} \frac{\left(2x^2+\frac{(2x^2)^3}{3!}\right)-\left(1+\frac{(x^3)^2}{2}\right)-\left(2x^2-\frac{(2x^2)^3}{3}\right)+1+o(x^6)}{x^6}$$

$$= \lim_{x\to 0} \frac{\cancel{2x^2}+\frac{8}{6}x^6-\cancel{1}-\frac{x^6}{2}-\cancel{2x^2}+\frac{8x^6}{3}+\cancel{1}+o(x^6)}{x^6}$$

$$= \lim_{x\to 0} \frac{\left(\frac{4}{3}-\frac{1}{2}+\frac{8}{3}\right)\cancel{x^6}}{\cancel{x^6}} = \frac{7}{2}.$$

L'ordine di infinitesimo del numeratore è 6.

3)
$$\begin{cases} (x-iy)^2-x-x-iy=0 \\ |z|\geq 1 \end{cases}$$

$$\begin{cases} x^2-y^2-2ixy-2x-iy=0 \\ |z|\geq 1 \end{cases}$$

$$\begin{cases} x^2-y^2-2x=0 \\ (2x+1)y=0 \\ x^2+y^2\geq 1 \end{cases}$$

$$\begin{cases} y=0 \\ x^2-2x=0 \\ x^2\geq 1 \end{cases} \cup \begin{cases} x=-\frac{1}{2} \\ y^2=\frac{1}{4}+1 \\ x^2+y^2\geq 1 \end{cases}$$

$$\begin{cases} y=0 \\ x=0 \\ |x|\geq 1 \end{cases} \cup \begin{cases} y=0 \\ x=2 \\ |x|\geq 1 \end{cases} \cup \begin{cases} x=-\frac{1}{2} \\ y=\pm\frac{\sqrt{5}}{2} \\ x^2+y^2\geq 1 \end{cases}$$

NO OK OK

Poiché $x^2+y^2=\frac{1}{4}+\frac{5}{4}=\frac{3}{2}\geq 1$

$$\Rightarrow z_1=(2;0)=2$$

$$z_{2,3}=\left(-\frac{1}{2};\pm\frac{\sqrt{5}}{2}\right)=-\frac{1}{2}\pm\frac{\sqrt{5}}{2}i$$

4) Con il criterio di convergenza assoluta:

$$\sum|a_n|=\sum\frac{1}{(2n+1)!}$$

$$\left|\frac{a_{n+1}}{a_n}\right|=\frac{1}{(2(n+1)+1)!}\cdot(2n+1)!$$

$$= \frac{1}{(2n+3)(2n+2)} \xrightarrow[n\to\infty]{} 0 < 1$$

$\Rightarrow$ la serie converge assolutamente e semplicemente.

<u>Altrimenti</u>, <u>col criterio di Leibniz</u>:

$$|a_n| = \frac{1}{(2n+1)!} \xrightarrow[n\to\infty]{} 0$$

$$|a_{n+1}| \le |a_n| \iff \frac{1}{(2n+3)!} \le \frac{1}{(2n+1)!}$$

$$\iff (2n+3)(2n+2)\cancel{(2n+1)!} \ge \cancel{(2n+1)!}$$

Passando ai reali:

$$(2x+3)(2x+2) \ge 1$$

$$4x^2 + 10x + 5 \ge 0$$

$$x_{1,2} = \frac{-5 \pm \sqrt{25-20}}{4} = \frac{-5 \pm \sqrt{5}}{4} < 0 \text{ entrambi}$$

~~Ma ci interessano~~

$$x \le x_1 \vee x \ge \underset{\underset{0}{\wedge}}{x_2}$$

Ma a noi interessano $n \geq 1$

Sempre vero $\forall n \geq 1$

$\Rightarrow$ sempre decrescente

$\Rightarrow$ serie convergente.

5) $$\int_1^{+\infty} \frac{\ln x}{x^3}\,dx = \left[-\frac{1}{2x^2}\ln x \Big|_1^{+\infty} + \int_1^{+\infty} \frac{1}{2x^3}\,dx \right]$$

$$= -\frac{1}{2}\Big[\underbrace{\lim_{x\to+\infty} \frac{\ln x}{x^2}}_{=0}\Big] + \frac{1}{2}\left[-\frac{1}{2x^2}\right]_1^{+\infty}$$

$$= -\frac{1}{4}\Big[\underbrace{\lim_{x\to+\infty} \frac{1}{x^2}}_{=0} - 1\Big] = \frac{1}{4}$$

La funzione è integrabile a $+\infty$ perché

~~perché~~ in $[1,+\infty)$ $0 \leq \frac{\ln x}{x^3} \leq \frac{x}{x^3} = \frac{1}{x^2}$ $\leftarrow$ integrabile a $+\infty$.