ONLINE SUPPLEMENTARY MATERIAL

http://www.dmmm.uniroma1.it/$\sim$bersani/mousetrap.html/tables.pdf

	$s = 2$	$s = 3$	$s = 4$
$m = 2$	6 4/4 (3/3) – SC immediate	20 7/7 (5/5) – SC immediate	70 10/10 (7/7) – SC immediate
$m = 3$	90 9/10 (5/5) – WC immediate	1680 16/16 (8/8) – SC immediate	34650 22/22 (11/11) – SC immediate
$m = 4$	2520 17/18 (7/7) – WC immediate	369600 28/28 (11/11) – SC immediate	63063000 38/38 (15/15) – SC immediate
$m = 5$	113400 27/28 (9/9) – WC immediate	168168000 43/43 (14/14) – SC $355,932$	$\sim 3.06 \cdot 10^{11}$ 58/58 (19/19) – SC $14,461,409$
$m = 6$	7484400 40/40 (11/11) – SC $4,530,195$	$\sim 1.37 \cdot 10^{11}$ 61/61 (17/17) – SC $123,289,316$	$\sim 3.25 \cdot 10^{15}$ 82/82 (23/23) – SC $314,429,118$
$m = 7$	681080400 54/54 (13/13) – SC $62,241,794$	$\sim 1.83 \cdot 10^{14}$ 82/82 (20/20) – SC $7,332,146,168$	$\sim 6.65 \cdot 10^{19}$ 110/110 (27/27) – SC $63,227,020,954$
$m = 8$	$\sim 8.17 \cdot 10^{10}$ 70/70 (15/15) – SC $4,152,727,936$	$\sim 3.69 \cdot 10^{17}$ 106/106 (23/23) – SC $\sim 147,000,000,000$	$\sim 2.39 \cdot 10^{24}$ 139/142 (31/31) – WC $\sim 264,386,000,000$
$m = 9$	$\sim 1.25 \cdot 10^{13}$ 88/88 (17/17) – SC $\sim 90,000,000,000$	$\sim 1.08 \cdot 10^{21}$ 131/133 (26/26) – WC $\sim 255,000,000,000$	$\sim 1.41 \cdot 10^{29}$ 172/178 (34/35) $> 207,000,000,000$
$m = 10$	$\sim 2.38 \cdot 10^{15}$ 106/108 (19/19) – WC $> 600,000,000,000$	$\sim 4.39 \cdot 10^{24}$ 154/163 (28/29) $> 81,000,000,000$	$\sim 1.29 \cdot 10^{34}$ 205/218 (37/39) $> 217,000,000,000$
$m = 11$	$\sim 5.49 \cdot 10^{17}$ 128/130 (21/21) – WC $92,800,000,000$	$\sim 2.39 \cdot 10^{28}$ 184/196 (31/32) $36,700,000,000$	$\sim 1.75 \cdot 10^{39}$ 224/262 (39/43) $2,000,000,000$
$m = 12$	$\sim 1.51 \cdot 10^{20}$ 139/154 (22/23) $12,000,000,000$	$\sim 1.71 \cdot 10^{32}$ 204/232 (33/35) $2,000,000,000$	$\sim 3.40 \cdot 10^{44}$ 273/310 (43/47) $1,000,000,000$
$m = 13$	$\sim 4.92 \cdot 10^{22}$ 158/180 (22/25) $2,000,000,000$	$\sim 1.56 \cdot 10^{36}$ 235/271 (34/38) $5,000,000,000$	$\sim 9.20 \cdot 10^{49}$ 305/362 (45/51) $4,000,000,000$

Table 1 - BEST SCORES IN $(HLM)^2N$ OBTAINED WITH MONTE CARLO METHODS

In each box of this table we report the number $N_{m \cdot s}$ of different decks; the ratio between the best score and C_{max}; the ratio between the best number of stored cards and the number predicted by (WC); the number of simulations performed before achieving the first winning deck or performed without obtaining any winning deck. The number of simulations is given by the sum of the trials done by F. Scigliano and by myself, while I have no information about the number of simulations done by A. Pompili. The symbols SC and WC indicate respectively if we proved the strong or the weak conjecture.

	$s=2$	$s=3$	$s=4$
$m=2$	$3/6$; $P_{max}=0.5$	$4/20$; $P_{max}=0.2$	$15/70$; $P_{max}\sim 0.21$
$m=3$	$0/90$; $P_{max}=0$	$4/1680$; $P_{max}\sim 0.0024$	$5/34650$; $P_{max}\sim 0.00014$
$m=4$	$0/2520$; $P_{max}=0$	$9/369,600$ $P_{max}\sim 0.000024$	$229/63,063,000$ $P_{max}\sim 0.0000036$
$m=5$	$0/113400$; $P_{max}=0$	$63/168,168,000$ $P_{max}\sim 0.000000375$	$10568/3.06\cdot 10^{11}$ $P_{max}\sim 0.000000035$
$m=6$	$1/7,484,400$ $P_{max}\sim 1.34\cdot 10^{-7}$	$1177/1.37\cdot 10^{11}$ $P_{max}\sim 0.000000009$	$1,212,483/3.25\cdot 10^{15}$ $P_{max}\sim 3.73\cdot 10^{-10}$
$m=7$	$7/681,080,400$ $P_{max}\sim 1.00\cdot 10^{-8}$	$36144/1.83\cdot 10^{14}$ $P_{max}\sim 1.98\cdot 10^{-10}$	$411,488,689/6.65\cdot 10^{19}$ $P_{max}\sim 6.19\cdot 10^{-12}$
$m=8$	$8/8.17\cdot 10^{10}$ $P_{max}\sim 9.79\cdot 10^{-11}$	$1,677,968/3.69\cdot 10^{17}$ $P_{max}\sim 4.54\cdot 10^{-12}$	
$m=9$	$105/1.25\cdot 10^{13}$ $P_{max}\sim 8.40\cdot 10^{-12}$	$127,255,522/1.08\cdot 10^{21}$ $P_{max}\sim 1.18\cdot 10^{-13}$	
$m=10$	$656/2.38\cdot 10^{15}$ $P_{max}\sim 2.76\cdot 10^{-13}$	$14,569,821,371/4.39\cdot 10^{24}$ $P_{max}\sim 3.32\cdot 10^{-15}$	
$m=11$	$6745/5.49\cdot 10^{17}$ $P_{max}\sim 1.23\cdot 10^{-14}$		
$m=12$	$76823/1.51\cdot 10^{20}$ $P_{max}\sim 5.07\cdot 10^{-16}$		
$m=13$	$986,994/4.92\cdot 10^{22}$ $P_{max}\sim 2.00\cdot 10^{-17}$		
$m=14$	$17,175,636/1.86\cdot 10^{25}$ $P_{max}\sim 9.23\cdot 10^{-19}$		
$m=15$	$320,152,788/8.09\cdot 10^{27}$ $P_{max}\sim 3.96\cdot 10^{-20}$		
$m=16$	$7,062,519,606/4.02\cdot 10^{30}$ $P_{max}\sim 1.76\cdot 10^{-21}$		

Table 2 - WINNING DECKS AT *HE LOVES ME HE LOVES ME NOT*

In each box we report the ratio between the number of winning decks and the total number of decks and the winning probability $P_{max}=P(C_{max})$.

	$s = 1$	winning deck(s)
$m = 2$	1/1 (1/1)	1 , 2
$m = 3$	3/4 (1/2 *and* 2/2)	three decks
$m = 4$	6/8 (3/3)	2 , 1 , 3 , 4
$m = 5$	9/13 (3/4)	2 , 5 , 1 , 4 , 3
$m = 6$	14/19 (4/5)	6 , 1 , 4 , 3 , 5 , 2
$m = 7$	18/26 (4/6)	3 , 7 , 1 , 5 , 2 , 6 , 4
$m = 8$	25/34 (5/7)	8 , 1 , 5 , 2 , 6 , 4 , 7 , 3
$m = 9$	31/43 (7/8)	4 , 1 , 2 , 6 , 9 , 7 , 3 , 8 , 5
$m = 10$	39/53 (6/9)	10 , 1 , 6 , 2 , 7 , 3 , 8 , 5 , 9 , 4
$m = 11$	47/64 (8/10 *and* 9/10)	six decks
$m = 12$	56/76 (7/11 *and* 10/11)	three decks
$m = 13$	67/89 (11/12)	two decks
$m = 14$	79/103 (12/13)	two decks
$m = 15$	93/118 (13/14)	two decks
$m = 16$	108/134 (14/15)	two decks
.....		

Table 3

In this table we report the ratio between the best score at $(HLM)^2N$ with one suit and C_{max} and the ratio between the number of stored cards and the number of cards satisfying (WC). In some cases it is possible to obtain the same best score with a different number of cards. When there is only one winning deck, we report it in the third column.

	$s=1$	$s=2$	$s=3$	$s=4$
$m = 2$	$1/2$; $P = 0.5$ [**G − N**]	$3/6$; $P = 0.5$	$4/20$; $P = 0.2$	$15/70$; $P \sim 0.21$
$m = 3$	$2/6$; $P \sim 0.33$ [**G − N**]	$12/90$; $P \sim 0.13$	$90/1680$; $P \sim 0.054$	$675/34650$; $P \sim 0.019$
$m = 4$	$6/24$; $P = 0.25$ [**G − N**]	$147/2520$; $P \sim 0.058$	$5232/369,600$ $P \sim 0.014$	$210,069/63,063,000$ $P \sim 0.0033$
$m = 5$	$15/120$ $P = 0.125$ [**G − N**]	$2322/113,400$ $P \sim 0.020$	$476,042/168,168,000$ $P \sim 0.0028$	$119,375,881/3.06 \cdot 10^{11}$ $P \sim 0.00039$
$m = 6$	$84/720$ $P \sim 0.12$ [**G − N**]	$71629/7,484,400$ $P \sim 0.0096$	$111,660,352/1.37 \cdot 10^{11}$ $P \sim 0.00081$	$P \sim 0.000070$ [**MC**]
$m = 7$	$330/5040$ $P \sim 0.065$ [**G − N**]	$2,214,258/681,080,400$ $P \sim 0.0033$	$P \sim 0.00016$ [**MC**]	$P \sim 0.0000081$ [**MC**]
$m = 8$	$1812/40320$ $P \sim 0.045$ [**G − N**]	$118,228,868/8.17 \cdot 10^{10}$ $P \sim 0.0014$	$P \sim 0.000046$ [**MC**]	$P \sim 0.0000015$ [**MC**]
$m = 9$	$9978/362,880$ $P \sim 0.027$ [**G − N**]	$6,597,279,578/1.25 \cdot 10^{13}$ $P \sim 0.00053$	$P \sim 0.000010$ [**MC**]	$P \sim 0.0000002$ [**MC**]
$m = 10$	$65503/3,628,800$ $P \sim 0.018$ [**C − S**]	$P \sim 0.00022$ [**MC**]	$P \sim 0.0000026$ [**MC**]	$P \sim 0.00000003$ [**MC**]
$m = 11$	$449,719/39,916,800$ $P \sim 0.011$ [**C − S**]	$P \sim 0.000083$ [**MC**]	$P \sim 0.0000006$ [**MC**]	$2 \cdot 10^{-9} < P < 6 \cdot 10^{-9}$ [**MC**]
$m = 12$	$3,674,670/479,001,600$ $P \sim 0.0077$ [**C − S**]	$P \sim 0.000036$ [**MC**]	$P \sim 0.000000084$ [**MC**]	$10^{-10} < P < 10^{-9}$ [**MC**]
$m = 13$	$28,886,593/6,227,020,800$ $P \sim 0.0046$ [**C − S**]	$P \sim 0.000013$ [**MC**]	$3 \cdot 10^{-8} < P < 5 \cdot 10^{-8}$ [**MC**]	$10^{-11} < P < 10^{-10}$ [**MC**]
$m = 14$	$266,242,729/8.72 \cdot 10^{10}$ $P \sim 0.0031$			
$m = 15$	$2,527,701,273/1.31 \cdot 10^{12}$ $P \sim 0.0019$			
$m = 16$	$25,749,021,720/2.09 \cdot 10^{13}$ $P \sim 0.0012$			

Table 4 - WINNING DECKS AT *MOUSETRAP*

In each box we report the ratio between the number of winning decks and $N_{m \cdot s}$ and the winning probability $P := P_{M,m \cdot s}(m \cdot s)$. We indicate with [**G-N**] and with [**C-S**] the results already quoted respectively in [**9**] and in [**4**], [**17**]. We indicate with [**MC**] the estimates obtained by means of Monte Carlo simulations.

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$m = 2$	$1/2$; $P = 0.5$ **[G-N]**	$5/6$; $P \sim 0.83$	$19/20$; $P = 0.95$	$69/70$; $P \sim 0.986$
$m = 3$	$4/6$; $P \sim 0.67$ **[G-N]**	$60/90$; $P \sim 0.67$	$1081/1680$; $P \sim 0.64$	$22898/34650$ $P \sim 0.66$
$m = 4$	$9/24$; $P = 0.375$ **[G-N]**	$1182/2520$; $P \sim 0.47$	$173,053/369,600$ $P \sim 0.47$	$29,642,185/63,063,000$ $P \sim 0.47$
$m = 5$	$76/120$; $P \sim 0.633$ **[G-N]**	$63063/113,400$ $P \sim 0.56$	$86,636,303/168,168,000$ $P \sim 0.52$	$P \sim 0.49$ **[MC]**
$m = 6$	$190/720$; $P \sim 0.26$	$1,797,350/7,484,400$ $P \sim 0.24$	$P \sim 0.23$ **[MC]**	$P \sim 0.22$ **[MC]**
$m = 7$	$3186/5040$; $P \sim 0.632143$	$364,572,156/681,080,400$ $P \sim 0.54$	$P \sim 0.49$ **[MC]**	$P \sim 0.46$ **[MC]**
$m = 8$	$11351/40320$ $P \sim 0.28$	$P \sim 0.24$ **[MC]**	$P \sim 0.22$ **[MC]**	$P \sim 0.21$ **[MC]**
$m = 9$	$132,684/362,880$ $P \sim 0.37$	$P \sim 0.31$ **[MC]**	$P \sim 0.28$ **[MC]**	$P \sim 0.27$ **[MC]**
$m = 10$	$884,371/3,628,800$ $P \sim 0.24$	$P \sim 0.20$ **[MC]**	$P \sim 0.18$ **[MC]**	$P \sim 0.18$ **[MC]**
$m = 11$	$25,232,230/39,916,800$ $P \sim 0.632120561$	$P \sim 0.53$ **[MC]**	$P \sim 0.48$ **[MC]**	$P \sim 0.45$ **[MC]**
$m = 12$	$50,436,488/479,001,600$ $P \sim 0.11$	$P \sim 0.085$ **[MC]**	$P \sim 0.077$ **[MC]**	$P \sim 0.073$ **[MC]**
$m = 13$	$3,936,227,868/6,227,020,800$ $P \sim 0.632120559$ [**A002467**]	$P \sim 0.53$ **[MC]**	$P \sim 0.48$ **[MC]**	$P \sim 0.45$ **[MC]**

Table 5 - WINNING DECKS AT *MODULAR MOUSETRAP*

In each box we report the ratio between the number of winning decks and $N_{m \cdot s}$ and the winning probability $P := P_{MM,m \cdot s}(m \cdot s)$. We indicate with **[G-N]** the results already quoted in [**9**].

The result corresponding to $m = 13$, $s = 1$ can be also obtained subtracting the total number of *derangements* to the total number of decks, $n! = m!$ (because m is prime). We indicate it with [**A002467**]. We indicate with **[MC]** the estimates obtained by means of Monte Carlo simulations.

	unreformed	1-reformed	2-reformed	3-ref.	4-ref.	5-ref.	1-cycles	total reformed
$m = 1$	0	0	0	0	0	0	1	1
$m = 2$	1	0	0	0	0	0	1	1
$m = 3$	4	2	0	0	0	0	0	2
$m = 4$	18	4	2	0	0	0	0	6
$m = 5$	105	14	1	0	0	0	0	15
$m = 6$	636	72	11	1	0	0	0	84
$m = 7$	4710	316	14	0	0	0	0	330
$m = 8$	38508	1730	81	1	0	0	0	1812
$m = 9$	$352,902$	9728	242	8	0	0	0	9978
$m = 10$	$3,563,297$	64330	1142	31	0	0	0	65503
$m = 11$	$39,467,081$	$444,890$	4771	56	2	0	0	$449,719$
$m = 12$	$475,326,930$	$3,645,441$	29009	219	1	0	0	$3,674,670$
$m = 13$	$6,198,134,207$	$28,758,111$	$127,876$	605	1	0	0	$28,886,593$
$m = 14$	$86,912,048,471$	$265,434,293$	$805,947$	2485	4	0	0	$266,242,729$
$m = 15$	$1,305,146,666,727$	$2,522,822,881$	$4,868,681$	9697	14	0	0	$2,527,701,273$
$m = 16$	$20,897,040,866,280$	$25,717,118,338$	$31,862,753$	40571	57	1	0	$25,749,021,720$

Table 6

Number of unreformed and reformed decks at *Mousetrap* for $s = 1$. The values for $1 \leq m \leq 9$ were reported by Guy and Nowakowski [**9**]. The values for $10 \leq m \leq 13$ were reported by Chua [**4**] and Sloane [**17**]. There is only one 5-reformed deck for $m = 16$. The first column extends the sequence [**17**] A007711; the second column extends [**17**] A007712; the third column extends [**17**] A055459; the fourth column extends [**17**] A067950; the last column extends [**17**] A007709; the sixth column corresponds to [**17**] A127966.

	unreformed	1-reformed	2-reformed	3-ref.	4-ref.	1-cycles	total reformed
$m = 1$	1	0	0	0	0	1	1
$m = 2$	3	2	0	0	0	1	3
$m = 3$	78	12	0	0	0	0	12
$m = 4$	2373	132	14	1	0	0	147
$m = 5$	$111,078$	2270	51	1	0	0	2322
$m = 6$	$7,412,771$	70766	857	6	0	0	71629
$m = 7$	$678,866,142$	$2,207,169$	7071	18	0	0	$2,214,258$
$m = 8$	$81,611,419,132$	$118,065,748$	$162,871$	249	0	0	$118,228,868$
$m = 9$	$12,498,038,864,422$	$6,593,940,635$	$3,337,216$	1723	4	0	$6,597,279,578$

Table 7

Number of unreformed and reformed decks at *Mousetrap* for $s = 2$. The case $m = 9$ yielded for the first time four 4-reformed deck.

	unreformed	1-reformed	2-reformed	3-reformed	1-cycles	total reformed
$m = 1$	0	0	0	0	1	1
$m = 2$	16	3	0	0	1	4
$m = 3$	1590	86	4	0	0	90
$m = 4$	$364,368$	5148	84	2	0	5232
$m = 5$	$167,691,958$	$474,658$	1384	1	0	$476,042$
$m = 6$	$137,113,427,648$	$111,570,619$	89649	84	0	$111,660,352$

Table 8

Number of unreformed and reformed decks at *Mousetrap* for $s = 3$. There is no evidence of 4-reformed decks in any case we have examined.

	unreformed	1-reformed	2-reformed	3-reformed	1-cycles	total reformed
$m = 1$	0	0	0	0	1	1
$m = 2$	55	11	4	0	1	15
$m = 3$	33975	639	35	0	1	675
$m = 4$	$62,852,931$	$209,411$	658	0	0	$210,069$
$m = 5$	$305,420,859,119$	$119,321,646$	54210	25	0	$119,375,881$

Table 9

Number of unreformed and reformed decks at *Mousetrap* for $s = 4$. In the case $m = 3$ we find for the first time a non-trivial 1-cycle: 111122322333. There is no evidence of 4-reformed decks, in any case we have examined.

	unreformed	k-reformed	cycles	total reformed
$m = 1$	0	0	1	1
$m = 2$	1	0	1	1
$m = 3$	2	2	2	4
$m = 4$	15	4	5	9
$m = 5$	44	37	39	76
$m = 6$	530	170	20	190
$m = 7$	1854	2336	850	3186
$m = 8$	28969	11077	274	11351
$m = 9$	$230,196$	$129,869$	2815	$132,684$
$m = 10$	$2,744,429$	$883,700$	671	$884,371$
$m = 11$	$14,684,570$	$21,529,972$	$3,702,258$	$25,232,230$
$m = 12$	$428,565,112$	$50,435,136$	1352	$50,436,488$
$m = 13$	$2,290,792,932$	3,456,154,665	480,073,203	$3,936,227,868$

Table 10

Number of unreformed and reformed decks at *Modular Mousetrap* for $s = 1$. The values for $1 \leq m \leq 5$ were reported by Guy and Nowakowski [**9**]. Since in this game, for $s = 1$ and m prime, a deck can only either win or give a *derangement*, we can obtain the number of unreformed decks by a theoretical point of view because it coincides with the number of *derangements* (see sequences [**17**] A000166 and A002467 and formula ( )).

	unreformed	k-reformed	cycles	total reformed
$m = 1$	0	0	1	1
$m = 2$	1	0	5	5
$m = 3$	30	39	21	60
$m = 4$	1338	1027	155	1182
$m = 5$	50337	57581	5482	63063
$m = 6$	$5,687,050$	$1,796,111$	1239	$1,797,350$
$m = 7$	$316,508,244$	$364,074,715$	$497,441$	$364,572,156$

Table 11

Number of unreformed and reformed decks at *Modular Mousetrap* for $s = 2$.

	unreformed	k-reformed	cycles	total reformed
$m = 1$	0	0	1	1
$m = 2$	1	0	19	19
$m = 3$	599	615	466	1081
$m = 4$	$196,547$	$161,772$	11281	$173,053$
$m = 5$	$81,531,697$	$86,339,122$	$297,181$	$86,636,303$

Table 12
Number of unreformed and reformed decks at *Modular Mousetrap* for $s = 3$.

	unreformed	k-reformed	cycles	total reformed
$m = 1$	0	0	1	1
$m = 2$	1	0	69	69
$m = 3$	11752	15466	7432	22898
$m = 4$	$33,420,815$	$29,381,680$	$260,505$	$29,642,185$

Table 13
Number of unreformed and reformed decks at *Modular Mousetrap* for $s = 4$.

	MAX k-reformed	MAX k-trajectory	MAX k-pre-period	MAX k-cycle	number of 1-cycles
$m = 1$	0	1	0	1	1
$m = 2$	0	1	0	1	1
$m = 3$	2	2	1	1	1
$m = 4$	2	3	2	1	1
$m = 5$	3	5	4	2	1
$m = 6$	5	5	4	1	1
$m = 7$	10	19	18	2	1
$m = 8$	8	9	8	2	1
$m = 9$	13	13	11	2	1
$m = 10$	10	6	5	3	1
$m = 11$	41	203	156	66	1
$m = 12$	8	7	6	1	1
$m = 13$	51	840	839	12	1
$m = 17$	≥ 51	≥ 39924	≥ 39923	≥ 209	≥ 1

Table 14

Longest sequences of deck reformations in the different cases (k-reformations, loops, pre-loops, k-cycles) at *Modular Mousetrap* for $s = 1$. In the last column we show the number of 1-cycles. For every value of m, the permutation $\{1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\}$ gives a 1-cycle. There is no evidence for other (non trivial) 1-cycles. In the case $m = 17$ we have examined only 50 million winning decks, because the total number of decks to be examined it too high. Since 17 is a prime number, it is highly probable that further investigation can improve the values we have up to now obtained.

	MAX k-reformed	MAX k-trajectory	MAX k-pre-period	MAX k-cycle	number of 1-cycles
$m = 1$	0	1	0	1	1
$m = 2$	0	3	2	1	2
$m = 3$	4	3	2	1	2
$m = 4$	9	7	5	2	2
$m = 5$	14	15	14	3	2
$m = 6$	13	7	6	2	2
$m = 7$	29	24	23	2	8

Table 15

Longest sequences of deck reformations in the different cases (k-reformations, loops, pre-loops, k-cycles) at *Modular Mousetrap* for $s = 2$. In the last column we report the number of 1-cycles. For every value of m, the permutation $\{1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\ ,\ 1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\}$ gives a 1-cycle. However, in this case we produced other (non trivial) 1-cycles.

	MAX k-reformed	MAX k-trajectory	MAX k-pre-period	MAX k-cycle	number of 1-cycles
$m = 1$	0	1	0	1	1
$m = 2$	0	4	2	2	2
$m = 3$	8	10	7	6	3
$m = 4$	17	12	10	2	5
$m = 5$	30	19	18	4	10

Table 16

Longest sequences of deck reformations in the different cases (k-reformations, loops, pre-loops, k-cycles) at *Modular Mousetrap* for $s = 3$. In the last column we report the number of 1-cycles. For every value of m, the permutation $\{1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\ ,\ \cdots\ 1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\}$ gives a 1-cycle. However, in this case we produced other (non trivial) 1-cycles.

	MAX k-reformed	MAX k-trajectory	MAX k-pre-period	MAX k-cycle	number of 1-cycles
$m = 1$	0	1	0	1	1
$m = 2$	0	5	3	3	3
$m = 3$	15	12	11	4	6
$m = 4$	28	17	14	3	10

Table 17

Longest sequences of deck reformations in the different cases (k-reformations, loops, pre-loops, k-cycles) at *Modular Mousetrap* for $s = 4$. In the last column we report the number of 1-cycles. For every value of m, the permutation $\{1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\ ,\ \cdots\ 1\ ,\ 2\ ,\ 3\ ,\ \cdots\ ,\ m-1\ ,\ m\}$ gives a 1-cycle. However, in this case we produced other (non trivial) 1-cycles.

	lowest value of n yielding a k-cycle	lowest value of n yielding a k-trajectory	lowest value of n yielding a k-reformed deck
$k = 1$	1	1	3
$k = 2$	5	3	3
$k = 3$	10	4	5
$k = 4$	11	5	6
$k = 5$	–	5	6

Table 18

Lowest value of n which produces a k-cycle, or a k-trajectory, or a k-reformed deck at *Modular Mousetrap*, with $s = 1$. The table is based on the complete results obtained for $m \leq 13$ and the partial results for $m = 17$. Let us observe that for $m = 11,\ 13$ we found even longer k-cycles, corresponding only to the values $k = 14, 15, 66$ for $m = 11$ and $k = 6, 7, 12$ for $m = 13$. For $m = 17$, up to now, we found only 1, 2, 170, 209-cycles.

The first value of n yielding k-trajectories, for $6 \leq k \leq 19$, is 7; the first value of n yielding k-trajectories, for $20 \leq k \leq 203$, is 11; the first value of n yielding k-trajectories, for $204 \leq k \leq 840$, is 13. Though in the case $m = 17$, we have only partial results, we know that 17 is the first value of n yielding at least all the k-trajectories for $841 \leq k \leq 39924$.

The first value of n yielding k-reformed decks, for $6 \leq k \leq 10$, is 7; the first value of n yielding k-reformed decks, for $11 \leq k \leq 13$, is 9; the first value of n yielding k-reformed decks, for $14 \leq k \leq 41$, is 11; the first value of n yielding k-reformed decks, for $42 \leq k \leq 51$, is 13.

	$s = 2$ (SC)	$s = 3$ (SC)	$s = 4$ (SC)
$m = 1$	$1 \quad 1 - cycle$ $1 \quad total\ reformed$	$1 \quad 1 - cycle$ $1 \quad total\ reformed$	$1 \quad 1 - cycle$ $1 \quad total\ reformed$
$m = 2$	$1 \quad 1 - cycle$ $2 \quad 1 - reformed$ $3 \quad total\ reformed$	$1 \quad 1 - cycle$ $3 \quad 1 - reformed$ $4 \quad total\ reformed$	$1 \quad 1 - cycle$ $10 \quad 1 - reformed$ $4 \quad 2 - reformed$ $15 \quad total\ reformed$
$m = 3$	$only\ unreformed$	$4 \quad 1 - reformed$ $4 \quad total\ reformed$	$5 \quad 1 - reformed$ $5 \quad total\ reformed$
$m = 4$	$only\ unreformed$	$9 \quad 1 - reformed$ $9 \quad total\ reformed$	$229 \quad 1 - reformed$ $229 \quad total\ reformed$
$m = 5$	$only\ unreformed$	$63 \quad 1 - reformed$ $63 \quad total\ reformed$	$10568 \quad 1 - reformed$ $10568 \quad total\ reformed$
$m = 6$	$1 \quad 1 - reformed$ $1 \quad total\ reformed$	$1177 \quad 1 - reformed$ $1177 \quad total\ reformed$	$1,212,483 \quad 1 - reformed$ $1,212,483 \quad total\ reformed$
$m = 7$	$7 \quad 1 - reformed$ $7 \quad total\ reformed$	$36144 \quad 1 - reformed$ $36144 \quad total\ reformed$	$411,488,689 \quad 1 - reformed$ $411,488,689 \quad total\ reformed$
$m = 8$	$8 \quad 1 - reformed$ $8 \quad total\ reformed$	$1,677,968 \quad 1 - reformed$ $1,677,968 \quad total\ reformed$	
$m = 9$	$105 \quad 1 - reformed$ $105 \quad total\ reformed$	$127,255,522 \quad 1 - reformed$ $127,255,522 \quad total\ reformed$	
$m = 10$	$656 \quad 1 - reformed$ $656 \quad total\ reformed$	$14,569,821,371 \quad 1 - reformed$ $14,569,821,371 \quad total\ reformed$	
$m = 11$	$6745 \quad 1 - reformed$ $6745 \quad total\ reformed$		
$m = 12$	$76823 \quad 1 - reformed$ $76823 \quad total\ reformed$		
$m = 13$	$986,994 \quad 1 - reformed$ $986,994 \quad total\ reformed$		
$m = 14$	$17,175,636 \quad 1 - reformed$ $17,175,636 \quad total\ reformed$		
$m = 15$	$320,152,788 \quad 1 - reformed$ $320,152,788 \quad total\ reformed$		
$m = 16$	$7,062,519,606 \quad 1 - reformed$ $7,062,519,606 \quad total\ reformed$		

Table 19

Number of reformed decks satisfying (SC) at $(HLM)^2N$. Since the value of P_{max} decreases very quickly when m grows, we cannot expect 2-reformed decks, apart from the case $m = 2$, $s = 4$.

	$s = 2$ (WC)	$s = 3$ (WC)	$s = 4$ (WC)
$m = 1$	$1\ 1-cycle$ $1\ total\ reformed$	$1\ 1-cycle$ $1\ total\ reformed$	$1\ 1-cycle$ $1\ total\ reformed$
$m = 2$	$1\quad 1-cycle$ $2\quad 1-reformed$ $3\quad total\ reformed$	$1\quad 1-cycle$ $4\quad 1-reformed$ $5\quad total\ reformed$	$1\quad 1-cycles$ $10\quad 1-reformed$ $4\quad 2-reformed$ $15\quad total\ reformed$
$m = 3$	$6\quad 1-reformed$ $6\quad total\ reformed$	$30\quad 1-reformed$ $30\quad total\ reformed$	$160\quad 1-reformed$ $160\quad total\ reformed$
$m = 4$	$10\quad 1-reformed$ $10\quad total\ reformed$	$278\quad 1-reformed$ $278\quad total\ reformed$	$7410\quad 1-reformed$ $1\quad 2-reformed$ $7411\quad total\ reformed$
$m = 5$	$56\quad 1-reformed$ $56\quad total\ reformed$	$5027\quad 1-reformed$ $5027\quad total\ reformed$	$669,948\quad 1-reformed$ $4\quad 2-reformed$ $669,952\quad total\ reformed$
$m = 6$	$200\quad 1-reformed$ $200\quad total\ reformed$	$132,437\quad 1-reformed$ $132,437\quad total\ reformed$	$133,085,352\quad 1-reformed$ $15\quad 2-reformed$ $133,085,367\quad total\ reformed$
$m = 7$	$1094\quad 1-reformed$ $1094\quad total\ reformed$	$6,131,753\quad 1-reformed$ $6,131,753\quad total\ reformed$	
$m = 8$	$7016\quad 1-reformed$ $7016\quad total\ reformed$	$436,816,134\quad 1-reformed$ $436,816,134\quad total\ reformed$	
$m = 9$	$55661\quad 1-reformed$ $55661\quad total\ reformed$		
$m = 10$	$586,810\quad 1-reformed$ $586,810\quad total\ reformed$		
$m = 11$	$7,340,841\quad 1-reformed$ $7,340,841\quad total\ reformed$		
$m = 12$	$114,616,993\quad 1-reformed$ $114,616,993\quad total\ reformed$		
$m = 13$	$2,030,647,546\quad 1-reformed$ $2,030,647,546\quad total\ reformed$		

Table 20

Number of reformed decks satisfying (WC) at $(HLM)^2N$. For $s = 4$, since the number of reformed decks grows very quickly, it is possible to find 2-reformed decks.

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