

Measure-valued Lagrangians

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Abstract

Measure-valued Lagrangians are presented here as an analytical tool allowing the construction of variational spaces of Sobolev type on non-differentiable structures with complicated geometry, like fractals. We first define Lagrangians on suitable subalgebras of the space of continuous functions - taking the role of the smooth C^1 functions of the classical case - and then proceed to the extension of the Lagrangian to spaces of possibly discontinuous functions. The extension is obtained by a *relaxation* procedure, which does not require the initial form to be *closable*. In the closable case, when the relaxed space coincides with the one obtained by completion, we also establish finer capacity properties of the Lagrangian.

In all, this theory can be seen as an attempt to build up a general analytical calculus for non-differentiable structures, possibly of fractal type. This theory can also be seen as providing a rigorous mathematical framework to the so-called *power-law* formalism adopted in physics for the description of scale invariant properties of various kind of fractal modes.¹

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