Waiting time phenomena for degenerate parabolic equations – a unifying approach

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Summary. We present a general criterion to establish the occurrence of waiting time phenomena for solutions to degenerate parabolic equations: Whenever a class of integral inequalities holds for a suitable choice of test–functions, a growth condition on the initial data is identified, which guarantees the existence of a positive time during which the support of solutions (locally in space) does not expand. This criterion applies both to the thin film equation and to doubly nonlinear degenerate parabolic equations of arbitrary order. Translating the growth condition into a "power–law–type" one, the critical exponent which we obtain coincides with the optimal one for the porous medium equation, and in higher space dimensions improves previous results for doubly nonlinear equations. The technique is based on Gagliardo–Nirenberg interpolation inequalities and on an extension of Stampacchia's lemma.