EVOLUTION AND MEMORY EFFECTS IN THE HOMOGENIZATION LIMIT FOR ELECTRICAL CONDUCTION IN BIOLOGICAL TISSUES

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ABSTRACT. We study an evolutive model for electrical conduction in biological tissues, where the conductive intracellular and extracellular spaces are separated by dielectric cellular membranes. The mathematical scheme is an elliptic problem, with dynamical boundary conditions of hyperbolic type on the cell membranes. The problem is set in a finely mixed periodic medium. We show that the homogenization limit u_0, obtained as the period of the microscopic structure approaches zero, solves the equation

$$-\operatorname{div}\left(\sigma_0\nabla_x u_0 + A^0\nabla_x u_0 + \int_0^t A^1(t-\tau)\nabla_x u_0(x,\tau)\,\mathrm{d}\tau - \mathcal{F}(x,t)\right) = 0\,,$$

where $\sigma_0 > 0$ and the matrices A^0, A^1 depend on geometric and material properties, while the vector function $\mathcal{F}$ keeps trace of the initial data of the original problem. Memory effects explicitly appear here, making this elliptic equation of non standard type.