Thin-film equations with "partial wetting" energy: existence of weak solutions ⋆

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Abstract

The capillarity driven evolution with slip of a liquid thin film over a dry surface is considered in the regime of "partial wetting". The focus is on the simplest model case of a constant, non-zero dynamic contact angle in the lubrication approximation. For the analytical treatment of the corresponding free boundary problem, a new strategy is proposed, based on the introduction of an ad hoc class of disjoining pressures which tend to concentrate at triple junctions. A first investigation of this approach yields to the existence of weak solutions which satisfy the dissipation relation as an inequality and which are different from those with zero contact angle. A heuristic argument is also presented in order to clarify the connection between contact angle and dissipation relation: it shows that moving droplets which satisfy the dissipation relation as an equality are forced to have the prescribed contact angle.

Key words: Liquid thin films, fourth order degenerate parabolic equations, free boundary problems
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