

INTRINSIC LOCALIZATION OF FRAMES

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ABSTRACT. Several concepts for the localization of a frame are studied. The intrinsic localization of a frame is defined by the decay properties of its Gramian matrix. Our main result asserts that the canonical dual frame possesses the same intrinsic localization as the original frame. The proof relies heavily on Banach algebra techniques, in particular on recent spectral invariance properties for certain Banach algebras of infinite matrices.

Intrinsically localized frames extend in a natural way to Banach frames for a class of associated Banach spaces which are defined by weighted ℓ^p -coefficients of their frame expansions. As an example the time-frequency concentration of distributions is characterized by means of localized (nonuniform) Gabor frames.

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Key Words: Frame, Banach frame, nonuniform Gabor frame, Banach *-algebras, spectral invariance, Gram matrix, pseudo-inverse, localization principle.

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