FLEXIBLE GABOR-WAVELET ATOMIC DECOMPOSITIONS FOR L^2-SOBOLEV SPACES

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ABSTRACT. In this paper we present a general construction of frames, which allows to ensure that certain families of functions (atoms) obtained by a suitable combination of translation, modulation and dilation form Banach frames for the family of L^2-Sobolev spaces on $\mathbb{R}$ of any order. In this construction a parameter $\alpha \in [0, 1)$ governs the dependence of the dilation factor on the frequency parameter. The well-known Gabor and wavelet frames (also valid for the same scale of Hilbert spaces) using suitable Schwartz functions as building blocks arise as special cases ($\alpha = 0$) and limiting case ($\alpha \to 1$) respectively. In contrast to those limiting cases it is no longer possible to use group theoretical arguments. Nevertheless we will show how to ensure *constructively* that for Schwartz analyzing atoms and any sufficiently dense, but discrete and well structured family of parameters one can guarantee the frame property. As a consequence of this novel constructive technique, one can generate quasi-coherent dual frames by an iterative algorithm. As will be shown in a subsequent paper the new frames introduced here generate Banach frames for corresponding families of α-modulation spaces.

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Key Words: Continuous/discrete frames, Gabor and wavelet frames, non-orthogonal expansions, α-modulation spaces, Sobolev spaces.

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