

# BANACH FRAMES FOR $\alpha$ -MODULATION SPACES

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**ABSTRACT.** This paper is concerned with the characterization of  $\alpha$ -modulation spaces by Banach frames, i.e., stable and redundant non-orthogonal expansions, constituted of functions obtained by a suitable combination of translation, modulation and dilation of a mother atom. In particular, the parameter  $\alpha \in [0, 1]$  governs the dependence of the dilation factor on the frequency. The result is achieved by exploiting intrinsic properties of localization of such frames. The well-known Gabor and wavelet frames arise as special cases ( $\alpha = 0$ ) and limiting case ( $\alpha \rightarrow 1$ ), to characterize respectively modulation and Besov spaces. This intermediate theory contributes to a further answer to the theoretical need of a common interpretation and framework between Gabor and wavelet theory and to the construction of new tools for applications in time-frequency analysis, signal processing, and numerical analysis.

**AMS subject classification:** 42B35, 42C15, 46B25, 65T60.

**Key Words:** Banach frames, Gabor analysis, localization of frames,  $\alpha$ -modulation spaces, wavelets.

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