ADAPTIVE FRAME METHODS FOR ELLIPTIC OPERATOR EQUATIONS

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Abstract. This paper is concerned with the development of adaptive numerical methods for elliptic operator equations. We are especially interested in discretization schemes based on frames. The central objective is to derive an adaptive frame algorithm which is guaranteed to converge for a wide range of cases. As a core ingredient we use the concept of Gelfand frames which induces equivalences between smoothness norms and weighted sequence norms of frame coefficients. It turns out that this Gelfand characteristic of frames is closely related to their localization properties. We also give constructive examples of Gelfand wavelet frames on bounded domains. Finally, an application to the efficient adaptive computation of canonical dual frames is presented.

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