

On a new class of multiresolution analyses generated by fractional refinable functions

Laura Pezza

Dipartimento Me. Mo. Mat.
Via Antonio Scarpa 16, 00161 Roma, Italy
pezza@dmmm.uniroma1.it

Abstract

A new class of refinable functions extending the GP class is presented. It is characterized by a symbol with fractional exponent that gives rise to non-compactly supported refinable functions. Nevertheless, the decay and the stability properties of these refinable functions allow them to generate a multiresolution analysis (MRA) of $L^2(\mathbb{R})$. For suitable values of their parameters these refinable functions reduce to the fractional B-splines, while, for integer α , they interpolate the GP refinable functions. Further, this class is proved to be closed with respect to the convolution and to the fractional differentiation, allowing to extend the applicability to Sobolev spaces. The fractional refinable functions introduced here show a surprising order of exactness.