On the posets $(\mathcal{W}_2^k, <)$ and their connections with some homogeneous inequalities of degree 2

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Abstract

A class of ranked posets $\{(D_h^k, \ll)\}$ has been recently defined in order to analyse, from a combinatorial viewpoint, particular systems of real homogeneous inequalities between monomials. In the present paper we focus on the posets D_2^k, which are related to systems of the form $\{x_a x_b *_{abcd} x_c x_d \colon 0 \leq a, b, c, d \leq k, *_{abcd} \in \{<, >\}, 0 < x_0 < x_1 < ... < x_k\}$. As a consequence of the general theory, the logical dependency among inequalities is adequately captured by the so-defined posets $(\mathcal{W}_2^k, <)$. These structures, whose elements are all the D_2^k's incomparable pairs, are thoroughly surveyed in the following pages. In particular, their order ideals - crucially significant in connection with logical consequence - are characterised in a rather simple way. In the second part of the paper, a class of antichains $\{\mathcal{P}_k \subseteq \mathcal{W}_2^k\}$ is shown to enjoy some arithmetical properties which make it an efficient tool for detecting incompatible systems, as well as for posing some compatibility questions in a purely combinatorial fashion.

Key-words: β-linearisation, compatible system, homogeneous inequality, logical consequence, monomial inequality, specular element.

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