

Some Remarks on Elliptic Problems with Critical Growth in the Gradient

Boumediene Abdellaoui*, Andrea Dall'Aglio†, Ireneo Peral* ‡

Abstract

In this work we analyze existence, nonexistence, multiplicity and regularity of solution to problem

$$\begin{cases} -\Delta u = \beta(u)|\nabla u|^2 + \lambda f(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where Ω is a bounded open set in \mathbb{R}^N , $\beta(s)$ is a positive continuous function, λ is a positive constant and $f(x)$ is a positive measurable function. We will also show a characterization of nonregular solutions of problem (1) as solutions of suitable semilinear elliptic problems with measure data.

⁰MSC2000, American Mathematical Society, 35D05, 35D10, 35J20, 35J25, 35J70, 46E30, 46E35.

Keywords: elliptic equations, existence and multiplicity.

*Departamento de Matemáticas, UAM, Campus de Cantoblanco, 28049 Madrid, Spain, email: boumediene.abdellaoui@uam.es, ireneo.peral@uam.es

†Dipartimento di Metodi e Modelli Matematici, Università di Roma *La Sapienza*, Via A. Scarpa, 16 I-00161, Italy, email: aglio@dmmm.uniroma1.it

‡First and third authors partially supported by Project BFM2001-0183. The second author was supported by project SAB2002-0156 MECD, Spain