

BANACH FRAMES FOR α -MODULATION SPACES II

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ABSTRACT. This paper is concerned with the characterization of α -modulation spaces by Banach frames, i.e., stable and redundant non-orthogonal expansions, constituted of functions obtained by a suitable combination of translation, modulation and dilation of a mother atom. In particular, the parameter $\alpha \in [0, 1]$ governs the dependence of the dilation factor on the frequency. The result is achieved by exploiting intrinsic properties of localization of such frames. The well-known Gabor and wavelet frames arise as special cases ($\alpha = 0$) and limiting case ($\alpha \rightarrow 1$), to characterize respectively modulation and Besov spaces. This intermediate theory contributes to a further answer to the theoretical need of a common interpretation and framework between Gabor and wavelet theory and to the construction of new tools for applications in time-frequency analysis, signal processing, and numerical analysis.

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