

# Continuous Frames, Function Spaces, and the Discretization Problem

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## Abstract

A continuous frame is a family of vectors in a Hilbert space which allows reproductions of arbitrary elements by continuous superpositions. Associated to a given continuous frame we construct certain Banach spaces. Many classical function spaces can be identified as such spaces. We provide a general method to derive Banach frames and atomic decompositions for these Banach spaces by sampling the continuous frame. This is done by generalizing the coorbit space theory developed by Feichtinger and Gröchenig. As an important tool the concept of localization of frames is extended to continuous frames. As a byproduct we give a partial answer to the question raised by Ali, Antoine and Gazeau whether any continuous frame admits a corresponding discrete realization generated by sampling.

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**Key Words:** continuous frames, discrete frames, coorbit spaces, function spaces, atomic decompositions, Banach frames, localization of frames, Banach algebras of kernels, general sampling methods

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