

**Abstract.** The aim of this work is to prove a  $L^1$ -lower semicontinuity theorem for integral functional defined on  $BV(\Omega)$ .

In particular, here, we obtain a lower semicontinuity result for the functional

$$\mathcal{F}(u) = \int_{\Omega} f(x, u, \nabla u) dx + \int_{\Omega} f^{\infty}(x, \tilde{u}, \frac{D^c u}{|D^c(u)|}) d|D^c u| + \int_{J_u} [\int_{u^-}^{u^+} f^{\infty}(x, s, \nu_u) ds] d\mathcal{H}^{N-1}$$

with respect to the  $L^1$ -convergence on  $BV(\Omega)$ . This theorem is stated by only requiring that the integrand  $f$  is  $W^{1,1}$  in  $x$  with a uniform control of the weak gradient and continuous in  $x$  (not uniformly with respect to the other variables).

The main tools of the proof of the lower semicontinuity theorem are a new chain rule formula for the function  $x \rightarrow \int_0^{u(x)} b(x, t) dt$  and an approximation result for convex functions due to De Giorgi.

Moreover we apply this result in order to obtain new relaxation and  $\Gamma$ -convergence result without any coerciveness and any continuity assumption of the integrand  $f(x, s, p)$  with respect to the variable  $s$ .