SPECTRAL ESTIMATES FOR RIEMANNIAN AND ALMOST RIEMANNIAN SUBMERSIONS WITH FIBERS OF BASIC MEAN CURVATURE

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Abstract.We estimate the eigenvalues of the Laplace-Beltrami operator Δ of the total space M of a Riemannian submersion whose fibers have basic mean curvature vector field. The first Sobolev space of M splits in the L^2-orthogonal direct sum of the subspaces of the functions constant on each fiber, resp. of zero integral on the fibers. As these subspaces are invariant for Δ, its spectrum is the union of the spectra of Δ itself restricted to them. The small eigenvalues are related to eigenfunctions constant on the fibers. The first non-zero eigenvalue has a lower bound depending on the geometry of the basis of the submersion and on the volume of the fibers; when all the fibers are minimal submanifolds of M, the dependance on their (constant) volume disappears. For an almost Riemannian submersion, we modify the horizontal part of the metric of M to get a new metric which makes Riemannian the submersion. The min-max and max-min principles give a pinching of the eigenvalues of the almost Riemannian submersion by the eigenvalues of the Riemannian one.