Nonlinear parabolic problems with a very general quadratic gradient term

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January 23, 2006

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Keywords: Nonlinear parabolic problems, gradient term with quadratic growth, convection-diffusion problems, superlinear reaction term, existence and regularity, bounded and unbounded solutions, a priori estimates, logarithmic Sobolev inequalities.

MSC2000: 35B33, 35B45, 35K20, 35K55, 35K57.

Abstract.

We study existence and regularity of distributional solutions for a class of nonlinear parabolic problems. The equations we consider have a quasi-linear diffusion operator and a lower order term, which may grow quadratically in the gradient and may have a very fast growth (for instance, exponential) with respect to the solution. The model problem we refer to is the following

$$\begin{cases} u_t - \Delta u = \beta(u)|\nabla u|^2 + f(x,t), & \text{in} \quad \Omega\times]0,T[; \\ u(x,t) = 0, & \text{on} \quad \partial\Omega\times]0,T[; \\ u(x,0) = u_0(x), & \text{in} \quad \Omega; \end{cases} \tag{1}$$

with $\Omega \subset \mathbb{R}^N$ a bounded open set, $T > 0$, and $\beta(u) \sim e^{|u|}$; as far as the data are concerned, we assume $\exp(\exp(|u_0|)) \in L^2(\Omega)$, and $f \in X(0,T;Y(\Omega))$, where X, Y are Orlicz spaces of logarithmic and exponential type, respectively. We also study a semilinear problem having a superlinear reaction term, problem that is linked with problem (1) by a change of unknown. Likewise, we deal with some other related problems, which include a gradient term and a reaction term together.