

Sailing towards, and then against,
the Graceful Tree Conjecture:
some promiscuous results

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Abstract

The Graceful Tree Conjecture is getting old – though 40 years are not so many – while researchers from all over the world keep on trying to put an affirmative end to it. Kotzig called a *disease* the effort of proving it. In this paper we fall into the opposite disease, namely by moving towards the search of a tree that is not graceful. Our first result, on suitable attachments of graceful trees, is however constructive and produces new graceful trees. But the reader might perceive a subtle friction between the combinatorial structure and the arithmetical need of producing a graceful labelling (that sensation will sound perhaps like a warning). Subsequently, the classification of all graceful labellings for a rather simple class of trees will seem at a first glance reassuring for its richness, while a more careful analysis may highlight some heavy constraints for labels, due to the mere structure of trees. Here the question is: what could happen to label constraints if the tree has a quite wild structure? Should we give up gracefulness? Finally, we introduce a polynomial associated to a given tree, which is expected to help the willing researcher to find some ungraceful tree, if any, in the next future.

Keywords: Graceful Tree Conjecture, Graceful Labelling.

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