SAMPLING THEOREMS ON BOUNDED DOMAINS

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Abstract. This paper concerns with iterative schemes for the perfect reconstruction of functions belonging to multiresolution spaces on bounded manifolds from nonuniform sampling. The schemes have optimal complexity in the sense that the computational cost to achieve a certain fixed accuracy is proportional to the computed quantity. Since the iterations converge uniformly, one can produce corresponding iterative integration schemes that allow to recover the integral of functions belonging to multiresolution spaces from nonuniform sampling. We present also an error analysis and, in particular, we estimate the L^2-error which one produces in recovering smooth functions in H^s, but not necessarily in any multiresolution space, and their integrals from nonuniform sampling. Several uni- and bi-variate numerical examples are illustrated and discussed. We also show that one can construct a rather large variety of multiresolution spaces on manifolds from certain refinable bases on the real line formed by so-called GP-functions. This class of functions that contains in particular B-splines has remarkable properties in terms of producing well-conditioned bases. The resulting multiresolution analyses are well-suited for the application of the iterative recovering of functions from nonuniform sampling.

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