

A BIZARRE GEOMETRIC CHARACTERIZATION OF ELLIPSES

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ABSTRACT. In this paper we prove that there are no domains $\mathcal{E} \subset \mathbb{R}^2$, other than the ellipses, such that the Lebesgue measure of the intersection of \mathcal{E} and its homothetic image $\varepsilon\mathcal{E}$ translated to a boundary point $q \in \partial\mathcal{E}$ is independent of q , provided that \mathcal{E} is “centered” at a certain interior point $O \in \mathcal{E}$ (the center of homothety).

Similar problems arise in various fields of mathematics, including, for example, the study of stationary isothermal surfaces and rearrangements.

KEYWORDS: Convex sets, asymptotic expansion, ordinary differential equations.

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