Nonlinear higher-order boundary value problems describing thin viscous flows near edges

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Abstract

Two boundary value problems for nonlinear higher-order ordinary differential equations are analyzed, which have been recently proposed to model steady, respectively quasi-steady, thin viscous flows over a bounded solid substrate. The first problem describes steady states and consists of a third-order ODE with an unknown parameter, the flux; boundary conditions prescribe, at the edges of the substrate, the height of the liquid and the tangential forces within the liquid as functions of the flux itself. For this problem, (non-)existence and non-uniqueness results are proved depending on the behavior, as the flux approaches zero, of the "height-function" (the height prescribed at the edge out of which the liquid flows). These results enforce the notion of "compatible" behaviors of the height-function as those for which a solution always exists. The second problem describes quasi-steady states and consists of a fourth-order ODE with nonlinear boundary conditions coupling the height and the flux: for this problem, we prove the existence of a solution for compatible behaviors of the height-function.

Key words: nonlinear boundary value problems (34B15); lubrication theory (76D08)

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