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A nonlocal and fully nonlinear degenerate parabolic system from strain-gradient plasticity

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Abstract

We consider a system of partial differential equations which describes anti-plane shear in the context of a strain-gradient theory of plasticity proposed by M.E. Gurtin [J. Mech. Phys. Solids 52 (2004), 2545-2568]. The problem couples a fully nonlinear degenerate parabolic system and an elliptic equation. It features two types of degeneracies: the first one is caused by the nonlinear structure, the second one by the dependence of the principal part on twice the curl of a planar vector field. Furthermore, the elliptic equation depends on the divergence of such vector field – which is not controlled by twice the curl – and the boundary conditions suggested by Gurtin are of mixed type.

To eliminate the latter complications we use a suitable, time-dependent representation of a divergence-free vector field which plays the role of the elastic stress. To handle the nonlinearities, by a suitable reformulation of the problem we transform the original system into one satisfying a monotonicity property which is more "robust" than the gradient flow structure inherited as an intrinsic feature of the mechanical model. These two insights make it possible to prove existence and uniqueness of a solution to the original system.