

# Noncommutative Korteweg-de Vries Hierarchy via Recursion Methods

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## Abstract

Here hierarchies of noncommutative nonlinear operator valued equations are studied. They represent a generalization to the operator level of corresponding hierarchies of noncommutative nonlinear evolution equations. Such hierarchies can, indeed, be obtained from the operator ones via a suitable projection. Key tool is the application of Bäcklund Transformations to relate different hierarchies of operator valued hierarchies of noncommutative nonlinear evolution equations. Indeed, in the case when hierarchies of nonlinear evolution equations in  $1 + 1$ -dimensions are considered, a wide Bäcklund Chart depicts all the links relating the Korteweg deVries (KdV) hierarchy of evolution equations, to the modified Korteweg deVries (mKdV) and Dym hierarchies. Notably, the algebraic structure, as well as the Hamiltonian and bi-Hamiltonian structure, for each equation, member of the Bäcklund Chart, is induced on combination of the well known properties of the KdV equation; such properties are proved when the whole hierarchy is considered on application of the hereditary property enjoyed by the recursion operator all these equations admit. Here the attention is focussed on the hierarchy of potential Korteweg deVries (pKdV) operator equations: the Bäcklund links with other hierarchies are exploited. Remarkably, hierarchies of noncommutative matrix, of any finite dimension, follow from this study, after a specific projection.