SUBDIVISION and REFINABILITY

workshop

Pontignano, Italy, May 1–4, 2008

Book of abstracts

PROGRAM

÷	**Thu. - May 1**	**Fri. - May 2**	**Sat. - May 3**	**Sun. - May 4**
8:45	Opening			
9:00	Goodman	Chui	Levin	Dubuc
10:00	Ron	Shen	Umlauf	Hormann
11:00	**coffee break**	**coffee break**	**coffee break**	**coffee break**
11:30 12:00	Unser	Sauer	Jetter	Romani Mößner
12:30				Closing
13:00	**lunch**	**lunch**	**lunch**	**lunch**
15:30	Sabin	Wallner	Han	
16:30	Cashman	Grohs	Charina	
17:00	**coffee break**	**coffee break**	**coffee break**	
17:30	Merrien	Yu	Cotronei	
18:30	Pitolli		Zimmermann	
19:00 20:00	**dinner**	**dinner** **trip to Siena**	**conference dinner**	

CONTENTS

Invited lectures

Charles K. Chui page 06

An MRA Approach to Surface Completion and Image Inpainting

Mariantonia Cotronei page 06

Full Rank Subdivision Schemes and Multichannel Wavelets

Serge Dubuc page 08

Subdivision Schemes and Seminormed Spaces

Tim Goodman page 08

Refinable Splines and Frames on Hybrid Meshes

Bin Han page 10

Symmetric Orthonormal Complex Wavelets and L_p Solutions of General Vector Refinement Equations

Kai Hormann page 11

Polynomial Reproduction by Symmetric Subdivision Schemes

Kurt Jetter page 12

Affine Combinations of Subdivision Schemes

David Levin page 13

Modified Subdivision Surfaces with Continuous Curvature

Jean Louis Merrien page 14

Hermite Subdivision: Convergence and Shape Preservation

Amos Ron page 18

Subdivision, Cascade and Refinable Functions: from Box Splines to L-CAMP

Malcolm Sabin page 20

The Analysis of Artifacts in Subdivision Curves and Surfaces

Tomas Sauer page 20

Shearlets and Subdivision – a Discrete Approach

Zuowei Shen page 21

A Tight Frame Approach for Missing Data Recovery in Images

Georg Umlauf page 21

Symmetry of Shape Charts with Applications to Energy-optimizing Subdivision Algorithms

Michael Unser page 22

Ten Good Reasons for Using Splines for Signal/Image Processing

Johannes Wallner page 23

New Results in Nonlinear Subdivision

Thomas Yu page 24

Smooth and Accurate Approximation of Manifold-Valued Data

Contributed talks

Tom Cashman page 05

Towards NURBS-Compatible Subdivision

Maria Charina page 05

Geometry Processing of Subdivision Surfaces

Philipp Grohs page 09

Interpolatory Wavelet Transforms in Manifolds

Bernhard Mößner page 15

C^1-Continuity of the Generalized Four-Point Scheme

Francesca Pitolli page 16

Bivariate Nonstationary Subdivision Schemes with Bell-shaped Limit Functions

Lucia Romani page 17

A General Approach towards the Construction of Tension-controlled 2ℓ-point Interpolatory Schemes with Salient Curves Reproduction Capabilities

Georg Zimmermann page 24

Polynomial Reproduction in Vector Subdivision: New Results on the Unstable Case

Speakers and Partecipants page 25

Towards NURBS-Compatible Subdivision

Tom Cashman, Neil Dodgson, Malcolm Sabin
Computer Laboratory, University of Cambridge, England
Numerical Geometry Ltd, Ely, England

Abstract

Non-Uniform Rational B-Spline (NURBS) surfaces can be non-uniform and defined for any degree, but existing subdivision surfaces are either uniform or of fixed degree. The resulting incompatibility forms a barrier to the adoption of subdivision for Computer-Aided Design. In this talk I will discuss our work towards a superset of NURBS which can handle irregularities in the control mesh. We call this target NURBS-compatible subdivision, as the resulting scheme would be able to represent any existing NURBS patch exactly. I will outline our strategy, discuss work which is already complete [1,2], and look at the challenges which lie ahead.

References

[1] Cashman, T. J., Dodgson, N. A., and Sabin, M. A., Non-uniform B-Spline Subdivision Using Refine and Smooth, In R. R. Martin, M. A. Sabin, and J. R. Winkler, editors. Volume **4647** of *Lecture Notes in Computer Science*. Springer (2007), pages 121–137.

[2] Cashman, T. J., Dodgson, N. A., and Sabin, M. A., A Symmetric, Non-uniform, Refine and Smooth Subdivision Algorithm for General Degree B-splines, *Computer Aided Geometric Design*, to appear (2008)

Geometry Processing of Subdivision Surfaces

Maria Charina, Joachim Stöckler
TU-Dortmund, Germany

Abstract

We present a local matrix factorization technique for constructing wavelet frame decompositions for subdivision surfaces. Such multiresolution analysis is one of the basic tools, e.g. for progressive compression of 3-dim meshes or interactive surface viewing. Our construction method employs results in [1] and techniques of semi-definite programming. We demonstrate our method on examples of interpolatory subdivision such as 4–point and butterfly schemes.

References

[1] C.K. Chui, W. He, J. Stöckler, *Nonstationary tight wavelet frames, I: Bounded Intervals*, ACHA, **17** (2004), pp. 141 - 197

An MRA Approach to Surface Completion and Image Inpainting

Charles K. Chui

University of Missouri, St. Louis, and Stanford University

Abstract

We will introduce a multi-resolution approximation (MRA) approach to the study of smooth function extensions with application to image inpainting. Solution of the Dirichlet problem relative to some Sturm-Liouville differential operator is used as the ground level of the MRA, and details in terms of anisotropic differential boundary data are filled in, according to the desirable number of higher-resolution levels. An error formula, in terms of the integral diffusion operators with the Greens functions of the anisotropic differential operators as diffusion kernels, is formulated and applied to derive the order of approximations.

Full Rank Subdivision Schemes and Multichannel Wavelets

Costanza Conti, **Mariantonia Cotronei**, Tomas Sauer

Dipartimento di Energetica ,Università di Firenze, Italy
DIMET, Università degli Studi "Mediterranea" di Reggio Calabria, Italy
Lehrstuhl für Numerische Mathematik, Justus–Liebig–Universität Gießen, Germany

Abstract

The *rank* of a *stationary vector subdivision scheme*, introduced in [6], yields a classification of subdivision schemes. In particular, the *full rank* condition assures that the scheme preserves *constant data*. If convergent, a full rank subdivision scheme admits a *matrix limit function* which is *refinable* with respect to the subdivision mask and which satisfies the *partition of identity* property. This function is the building block of a *multichannel multiresolution analysis* (*MMRA*) and of a corresponding *multichannel wavelet*, which represent the most suitable "wavelet" tools for the analysis of *vector-valued signals*. In particular, like in the scalar and multiwavelet case, it can be proved that the existence of a matrix refinable function with orthonormal integer translates guarantees the existence

of a matrix multichannel wavelet function with orthonormal integer translates as well.

We are mainly interested in *full rank interpolatory schemes*, and the reasons can be summarized as follows:

- full rank appears most naturally in the context of *interpolatory vector subdivision schemes*, that is, if such schemes are convergent, then they must necessarily be full rank, and the associated basic limit function is *cardinal*;
- a vector interpolatory scheme naturally gives rise to an MMRA of interpolatory type;
- there is a connection between cardinal and orthogonal refinable functions which is based on the *spectral factorization* of the (positive definite) symbol related to the cardinal function, thus, as a by-product, giving a concrete way to obtain orthogonal matrix scaling functions and multichannel wavelets.

In this talk we review all the results jointly obtained on full rank interpolatory subdivision schemes and we discuss properties and convergence issues of several different types of refinable function which arise in this context. Moreover, we give explicit construction of MMRAs and multichannel wavelets together with some examples of application.

References

[1] S. Bacchelli, M. Cotronei, T. Sauer, Wavelets for multichannel signals, *Advances Appl. Math.*, **29**, 581–598, (2002).

[2] Full rank interpolatory subdivision schemes: Kronecker, filters and multiresolution, submitted for publication.

[3] C. Conti, M. Cotronei, T. Sauer, Full rank positive matrix symbols: interpolation and orthogonality, *BIT*, **48**, 5-27 (2008).

[4] C. Conti, M. Cotronei, T. Sauer, Interpolatory vector subdivision schemes, in: A. Cohen, J. L. Merrien, L. L. Schumaker (eds.), Curves and Surfaces, Avignon 2006, Nashboro Press, (2007).

[5] M. Cotronei, T. Sauer, Full rank filters and polynomial reproduction, *Comm. Pure Appl. Anal.*, **6**, 667–687, (2007).

[6] C. A. Micchelli, T. Sauer, Regularity of multiwavelets, *Advances Comput. Math.*, **7 (4)**, 455–545, (1997).

Subdivision Schemes and Seminormed Spaces

Serge Dubuc

Department of Mathematics and Statistics, University of Montreal, Canada

Abstract

We discuss three criteria of convergence of subdivision schemes for curves. The first one has been found by Gregory, Dyn and Levin [3] in the uniform case, and by Buhmann and Micchelli [1] in the general case. The second one, by Daubechies and Lagarias [2], covers only the uniform case. The third one is proposed by the author. We relate these three criteria together by using specific seminorms for subdivision operators, seminorms that make them equivalent. In particular, the last two criteria are linked through the duality theory.

References

[1] M. D. Buhmann and C. Micchelli, Using two-slanted matrices for subdivision. *Proc. London Math. Soc. (3)* **69** (1994), 428-448.

[2] I. Daubechies and J. C. Lagarias, Two-scale difference equations II. Local regularity, infinite products of matrices and fractals. *SIAM J. Math. Anal.* **22** (1992) 1031-1079.

[3] N. Dyn, J.A. Gregory and D. Levin, Analysis of linear binary subdivision schemes for curve design, *Constr. Approx.* **7** (1991), no. 2, 127-147.

Refinable Splines and Frames on Hybrid Meshes

Tim Goodman

University of Dundee, Scotland, UK

Abstract

We consider refinable spaces of spline functions on meshes which are uniform away from the origin and geometric near the origin, thus allowing more detailed analysis near the origin, which is regarded as a point of special interest (or *fovea*). In the multivariate case, the space is not a tensor-product space but it is spanned by tensor-product B-splines. Orthogonal decomposition over different scales is considered and it is shown that in general there is no Riesz basis of compactly supported prewavelets. In contrast a simple construction is given of a tight frame of compactly supported functions. Some generalisations to non-spline spaces are also considered.

Interpolatory Wavelet Transforms in Manifolds

Philipp Grohs, Johannes Wallner

Institute of Geometry, TU Graz. Kopernikusgasse 24, A 8010 Graz, Austria

Abstract

The topic of the talk will be the recent work [1] on interpolatory wavelet transforms in manifolds. We discuss how to define such transforms in a general framework. Then, extending a result of Donoho [2] to the manifold-valued setting, we show how the decay rate of the wavelet coefficients is related to the smoothness of the corresponding function.

References

[1] Philipp Grohs and Johannes Wallner, *Interpolatory wavelets for manifold-valued data*, preprint, TU Graz, March 2008.

[2] David L. Donoho, *Interpolating wavelet transforms*, preprint, 1992.

Symmetric Orthonormal Complex Wavelets and L_p Solutions of General Vector Refinement Equations

Bin Han

Department of Mathematical and Statistical Sciences,
University of Alberta, Edmonton, Alberta, Canada T6G 2G1.

Abstract

This talk consists of two parts. In the first part of the talk, we shall present a comprehensive study of (dyadic) orthonormal complex wavelets with symmetry. We further show that dyadic orthonormal complex wavelets with symmetry can be easily constructed such that they can have arbitrarily high smoothness or arbitrarily high order of coiflet property. This is also true for other dilations such as dilation 4. Then we demonstrate that there are some interesting connections between orthonormal complex wavelets, tight multiwavelet frames, and vector refinement equations of full rank. In the second part of the talk, we shall consider the L_p solution ϕ to the following general refinement equation:

$$\phi(x) = |\det M| \sum_{k \in Z^s} a_k \phi(Mx - k), \qquad x \in R^s,$$

where M is a general $s \times s$ dilation matrix and $a := \{a_k\}_{k \in Z^s}$ is a finitely supported mask with each a_k being an $r \times r$ matrix of complex numbers. We impose no conditions on the mask a and the dilation matrix M. We do not require that 1 should be a simple eigenvalue of the matrix $\sum_{k \in Z^s} a_k$. The convergence of the vector subdivision scheme in the Sobolev space $W_p^k(R^s)$ and smoothness of the refinable function ϕ will be discussed. For example, as a consequence, for $1 \le p \le \infty$, we show that if $\phi \in L_p(R^s)$ (when $p = \infty$, replace $L_p(R^s)$ by $C(R^s)$) is a compactly supported function satisfying the above refinement equation, then ϕ must belong to some Lipschitz class with a positive Lipschitz exponent.

References

[1] B. Han, Symmetric complex Coiflets of arbitary orders, preprint, (2007).

[2] B. Han and H. Ji, Compactly supported orthonormal complex wavelets with dilation 4 and symmetry, preprint, (2008).

[3] B. Han, Solutions in Sobolev spaces of vector refinement equations with a general dilation matrix, *Adv. Comput. Math.* **24** (2006), 375–403.

Polynomial Reproduction by Symmetric Subdivision Schemes

Nira Dyn, **Kai Hormann**, Malcolm Sabin, Zuowei Shen

Tel Aviv University
Clausthal University of Technology
University of Cambridge
National University of Singapore

Abstract

This talk is about some new insights regarding the reproduction of polynomial functions by a linear, binary, uniform, and stationary subdivision scheme. While the necessary and sufficient conditions for such a scheme to *generate* polynomials of a certain degree as limit functions are well understood, little is known about polynomial *reproduction* in the sense that taking uniformly spaced samples from a polynomial as the initial data of the subdivision scheme yields the same polynomial in the limit. So far, conditions for this kind of reproduction are known only for interpolating schemes, although this property is very important as it is directly connected to the approximation order of the subdivision scheme.

We first present necessary and sufficient conditions for a subdivision scheme to have polynomial reproduction of degree d and thus approximation order $d+1$. Our conditions are partly algebraic and easy to check by considering the symbol of a subdivision scheme, but also relate to the parameterization of the scheme. After discussing some special properties that hold for symmetric schemes, we then use our conditions to derive the maximum degree of polynomial reproduction for two families of symmetric schemes, the family of pseudo-splines and a new family of dual pseudo-splines.

References

[1] N. Dyn, K. Hormann, M. A. Sabin, and Z. Shen, Polynomial reproduction by symmetric subdivision schemes, Technical Report IfI-07-13, Department of Informatics, Clausthal University of Technology (2007).

[2] K. Hormann and M. A. Sabin, A family of subdivision schemes with cubic precision, *Computer Aided Geometric Design*, **25**, 41–52 (2008)

Affine Combinations of Subdivision Schemes

Costanza Conti and **Kurt Jetter**

Università degli Studi di Firenze
Universität Hohenheim

Abstract

Convex and affine combinations of subdivision schemes have been used frequently in the study of stationary or non-stationary subdivision, in particular for smoothing or fairing the limit functions, or for the purpose of increasing their regularity. In this talk we will take a rather general approach, starting with a set of d-variate, scalar or vector subdivision schemes with masks $\mathrm{A}^{(i)} = (\mathrm{a}_\alpha^{(i)})_{\alpha\in\mathbb{Z}^d}$ which we assume to be finitely supported. The corresponding mask symbols

$$\mathrm{a}^{(i)}(z) = \sum_\alpha \mathrm{a}_\alpha^{(i)} z^\alpha ,$$

are then combined according to

$$\mathrm{c}(z) = \sum_{i=1}^n \lambda_i \, z^{\sigma_i} \, \mathrm{a}^{(i)}(z) , \quad \sum_{i=1}^n \lambda_i = 1 ,$$

where $\lambda_i \in \mathbb{R}$ are given weights, and $\sigma_i \in \mathbb{Z}^d$ are given shifts.

We shall discuss and list some properties (such as contractivity, or polynomial reproduction, etc.) which are kept under such combinations, and which may be improved by choosing the weights and the shifts appropriately. Examples of such combinations from the literature will refer to the references below.

We will discuss the stationary case only, where our discussion is rather complete for the scalar case, and for the full-rank vector case.

References

[1] C. Conti and K. Jetter, Smoothed hat functions in subdivision, *J. Comp. Appl. Math.* (2007), xxx-xxx.

[2] N. Dyn, D. Levin and C. A. Micchelli, Using parameters to increase smoothness of curves and surfaces generated by subdivision, *Comp. Aided Geometric Design* **7** (1990), 129–140.

[3] L. Gori and F. Pitolli, A class of totally positive refinable functions, *Rendiconti di matematica, ser. VII*, Roma, **20** (2000), 305–322.

Modified Subdivision Surfaces with Continuous Curvature

Adi Levin, **David Levin**

Tel Aviv University, Israel

Abstract

We present a modification to subdivision surfaces, which guarantees second-order smoothness everywhere in the surface, including extraordinary points. The idea is to blend the limit surface with a low degree polynomial defined over the characteristic map, in the vicinity of each extraordinary point. We demonstrate our method on Catmull-Clark surfaces, but a similar modification can be applied to other schemes as well. The proposed modification to Catmull-Clark is simple to implement and can be applied to quad meshes of arbitrary topological type, even when extraordinary vertices share edges.

Hermite Subdivision: Convergence and Shape Preservation

Jean-Louis Merrien

INSA de Rennes, France

Abstract

In the past fifteen years, Hermite subdivision has been studied by different authorss. The advantage of this algorithm is that the construction is usually very local once the exact or approximate derivatives of a function are known. The construction of the functions can be stopped as soon as they are defined on enough points to get a nice design on a screen.

But the convergence is also a proper question. Some tools are well known for vector subdivision schemes and different methods have been proposed to transform Hermite subdivision schemes into vector or scalar Subdivision schemes or to compare them (see [2, 3, 4, 5]).

More recently, these Hermite schemes have been used for interpolation with shape preserving properties. The method leads to the use of control polygons (or grid) and, in dimension 1, we can find a Bernstein-like basis and we can also use total positivity (see [1, 6, 7]).

References

[1] P. Costantini and C. Manni, On Constrained Nonlinear Hermite Subdivision, to appear in *Const. Approx.*

[2] S. Dubuc, J.-L. Merrien, Hermite Subdivision Schemes and Taylor Polynomials, to appear in *Const. Approx.*

[3] N. Dyn, D. Levin, Analysis of Hermite-interpolatory subdivision schemes. In *Spline Functions and the Theory of Wavelets*, S. Dubuc et G. Deslauriers, Ed. Amer. Math. Soc., Providence R. I., 1999, pp. 105-113.

[4] B. Han, Vector cascade algorithms and refinable functions in Sobolev spaces, *J. Approx. Theory* **124** (2003), 44-88.

[5] B. Han, T. Yu and Y. Xue, Noninterpolatory Hermite subdivision schemes, *Math. Comp.* **74** (2005), 1345-1367.

[6] T. Lyche, J.-L. Merrien, C1 Interpolatory Subdivision with Shape Constrains for Curves, *Siam J. Numer. Anal.*, **44**, 1095 - 1121 (2006),

[7] T. Lyche, J.-L. Merrien, C1 Hermite Subdivision with Shape Constrains on a Rectangular mesh, *BIT Num Math*, **46**, 831-859, (2006),

C^1-Continuity of the Generalized Four-Point Scheme

Jochen Hechler, **Bernhard Mößner**, Ulrich Reif

TU Darmstadt, Schlossgartenstr. 7, 64289 Darmstadt, Germany
Universität Freiburg, Hermann-Herder-Str. 10, 79104 Freiburg, Germany
TU Darmstadt, Schlossgartenstr. 7, 64289 Darmstadt, Germany

Abstract

The interpolatory four-point subdivision scheme was introduced in 1986 by Dubuc [1], and generalized in 1987 by means of a real tension parameter ω by Dyn et al. [2]. Several technics for testing differentiability of subdivision-schemes have been developed, but determining the exact bounds $0 < \omega < \omega^*$ for which the generalized four-point scheme generates C^1-limits still remained an open problem.

Following the matrix-approach by Micchelli and Prautzsch [3] those ω have to be determined for which the joint spectral radius of two (4×4)-matrices A_0, A_1 is less the 1. We show that $\operatorname{jsr}(A_0, A_1) < 1$, iff $0 < \omega < \omega^*$, where $\omega^* \approx 0.19273$ is the unique real solution of the equation $32\omega^3 + 4\omega - 1 = 0$.

References

[1] S. Dubuc, *Interpolation through an iterative scheme*, Journal of Mathematical Analysis and Applications, Vol. 114(1) (1986), pp. 185-204.

[2] N. Dyn, J.A. Gregory and D. Levin, *A 4-point interpolatory subdivision scheme for curve design*, Computer Aided Geometric Design, Vol. 4(4) (1987), pp. 257-268.

[3] C.A. Micchelli and H. Prautzsch, *Uniform refinement of curves*, Linear Algebra and its Applications, Vol. 114/115 (1989), pp. 841-870.

Bivariate Nonstationary Subdivision Schemes with Bell-shaped Limit Functions

Francesca Pitolli
Dept. Me.Mo.Mat., Università di Roma "La Sapienza", Italy

Abstract

A sequence of *refinement masks* $\{\mathbf{a}^k\}_{k\geq 0}$ can be associated to a *nonstationary subdivision scheme* $S_{\{\mathbf{a}^k\}}$ as follows [1], [4]:

$$\lambda^0 := \lambda \in \ell(\mathbb{Z}^s), \quad \lambda^{k+1} := S_{\mathbf{a}^k}\ \lambda^k, \qquad k \geq 0,$$

where $S_{\mathbf{a}^k} : \ell(\mathbb{Z}^s) \to \ell(\mathbb{Z}^s)$, $(S_{\mathbf{a}^k}\ \lambda)_\alpha := \sum_{\beta\in\mathbb{Z}^s} a^k_{\alpha-2\beta}\ \lambda_\beta$, $\alpha \in \mathbb{Z}^s$, is the *k-level subdivision operator*. In the following we assume $\mathbf{a}^k$, $k \geq 0$, to have compact support. In case of convergence, there exists a family of *basic limit functions* ϕ_m, defined as $\phi_m := S^\infty_{\{\mathbf{a}^k\}_{k\geq m}}\ \delta_0$, $m \geq 0$, that are solutions to the *nonstationary refinement equations* [4],[5]

$$\phi_m = \sum_{\alpha\in\mathbb{Z}^s} a^m_\alpha\ \phi_{m+1}(2\cdot - \alpha), \quad m \geq 0. \tag{1}$$

Nonstationary subdivision schemes allow to obtain basic limit functions with small support, since it is related to the starting mask, and high smoothness, which is given by the limit mask [4]. In the univariate case a well known example is the up-function, which is compactly supported on $[0, 2]$ and belongs to C^∞. Actually, the up-function can be viewed as the basic limit function ϕ_0 of a nonstationary subdivision scheme related to the binomial masks [1]. Other examples of univariate nonstationary refinable functions with small support and high smoothness can be found in [2], [6].

We present a new class of bell-shaped bivariate refinable functions, obtained by a suitable use of certain bivariate subdivision schemes [3]. These functions are characterized by having small support and their smoothness can be established a priori.

References

[1] Cohen, A., and N. Dyn, Nonstationary subdivision schemes and multiresolution analysis, *SIAM J. Math. Anal.*, **27**, 1745–1769 (1996).

[2] C. Conti, L. Gori, F. Pitolli, Totally positive functions through nonstationary subdivision schemes, *J. Comp. Math.*, **200**, 255–265 (2007).

[3] C. Conti, F. Pitolli, A new class of bivariate refinable functions suitable for cardinal interpolation, *Rend. Mat. Appl. VII*, **27**, 61-71 (2007).

[4] N. Dyn, D. Levin, Subdivision schemes in geometric modelling, *Acta Numerica*, **11**, 73–144 (2002).

[5] T.N.T. Goodman, S.L. Lee, Convergence of nonstationary cascade algorithms, *Numer. Math.*, **84**, 1–33 (1999).

[6] L. Gori, F. Pitolli, Nonstationary subdivision schemes and totally positive refinable functions, in Approximation Theory XII: San Antonio 2007, M. Neamtu and L.L. Schumaker eds., Nashboro Press, (2008), 169–180.

A General Approach towards the Construction of Tension-controlled 2ℓ-point Interpolatory Schemes with Salient Curves Reproduction Capabilities

Lucia Romani

Department of Mathematics and Applications, University of Milano-Bicocca, Italy

Abstract

This work is concerned with the description of a general approach for the construction of 2ℓ-point interpolatory schemes exhibiting two properties of great interest in curve design: tension control and salient curves reproduction.
Based on the observation that the Laurent polynomial formalism of approximating schemes whose limit functions belong to subclasses of L-splines [2] can be derived straightforwardly, we will present an algorithm that, taken as input the Laurent polynomial representation of an approximating scheme reproducing functions in a given space of exponential polynomials, will provide the Laurent polynomial representation of a 2ℓ-point interpolating scheme with the same reproduction properties [1].
The proposed algorithm will be applied to define three novel 6-point interpolating schemes that turn out to be unique in combining the capability of generating $C2$-continuous limit curves with the attractive properties of tension control and important analytical shapes reproduction.
The introduced interpolatory 6-point $C2$ subdivision schemes will be shown to be different non-stationary versions of the stationary proposal in [3].

References

[1] Dyn, N., Levin, D., Luzzatto, A., Exponentials reproducing subdivision schemes, *Found. Comput. Math.*, **3**, 187-206 (2003)

[2] Schumaker, L.L., Spline functions: basic theory, Wiley, New York, (1981)

[3] Weissmann, A., A 6-point interpolating subdivision scheme for curve design, M.Sc. Thesis, Tel-Aviv University, (1990)

Subdivision, Cascade and Refinable Functions: from Box Splines to L-CAMP

Amos Ron

Computer Science Department
University of Wisconsin – Madison

Abstract

My presentation will be divided equally between two topics that are entirely disjoint, still fall, each, squarely within the scope of the meeting.

In the first one, I will discuss the notion of speed of convergence of cascade and subdivision algorithms. The fundamental rule is that the speed of convergence of such algorithms cannot exceed a natural limit that is imposed by the smoothness of the limit, viz. the refinable function. The second rule, which goes back to [1], shows that this natural, maximal, speed of convergence may not be attainable. Indeed, we all know of very smooth refinable functions for which the basic subdivision algorithm does not converge at all. My general goal in this area is to develop a theory and that will identify the reasons for this degraded speed of convergence. The basic tool to this end is known that the *Double Tree Theorem*, [1], that intertwines the cascade operator and the subdivision operator, and reduces the problem to the understanding of the action of the subdivision operator on the space of dependence relations of the refinable function. However, the latter space is infinite dimensional in general (unless we are in one dimension, which is a trivial case as far my stated goal above is concerned), which challenges the algorithmic implementation of the theory.

In [2], a complete solution to the above problem is given in the case the refinable function is a box spline. We handle the box spline case by showing first that the special structure of the subdivision mask in this case allows us the examine the action of the subdivision operator on a small finite dimensional subspace of the (possibly infinite dimensional) space of dependence relations. We then settle the problem completely, and show that the solution is completely in terms of rudimentary linear algebra terms: not the incomputable joint spectral radius, and not even any (computable, but unnecessary) spectral radius of anything. This part is joint with Carl de Boor (Madison and Seattle).

In the second part of the talk, after spending a few minutes on the topic of the L-CAMP representation, [3], I will introduce the class of refinable functions that appear in that theory. A typical refinable function there is in 2-6 dimensions, and with a non-separable mask. The greatness of the L-CAMP representation is that the above complicated mask never appears in the actual algorithms (viz. decomposition and reconstruction), since the L-CAMP algorithms bypass the Fast Wavelet transform. However, we must estimate the

smoothness of the refinable function, since this is important for the understanding of the performance of the L-CAMP representation. I will briefly discuss the techniques that we developed for the estimation of the smoothness of this class of refinable functions. This part is joint work with my former student Youngmi Hur (MIT) and my current student Sangnam Nam (Madison).

References

[1] M. Neamtu, A. Ron and Zuowei Shen, The cascade and subdivision algorithms, ms. 27 July, (1998).

[2] Carl de Boor and Amos Ron, Box splines revisited: convergence and acceleration methods for the subdivision and the cascade algorithms, *Journal of Approximation Theory* **150**, 1–23 (2008).

[3] Youngmi Hur and Amos Ron, L-CAMP: extremely local high-performance wavelet representations in high spatial dimension, *IEEE Transactions on Information Theory* **xx**, xxx-xxx (200x).

The Analysis of Artifacts in Subdivision Curves and Surfaces

Malcolm Sabin
Computer Laboratory, Cambridge University, UK

Abstract

Any scheme which creates curves or surfaces from control points may suffer from artifacts: that is '*features which appear in the final curve or surface, which cannot be removed by movement of the control points*'. It turns out that the size and nature of these can be analysed fairly straightforwardly when the scheme is expressed by a subdivision construction. Although the initial motivation was certain artifacts which appear round extraordinary vertices of excessive valency in surface schemes, there is a good foundation in the univariate theory, which relates the size of the artifacts found to a simple transformation of the symbol of a stationary scheme. This extends naturally to surfaces over both quad- and triangular- grids. It can also be used to deliberately construct non-stationary schemes which have zero artifact for data at specific spatial frequencies.

Shearlets and Subdivision – a Discrete Approach

Gitta Kutyniok, **Tomas Sauer**
Stanford University, USA
Lehrstuhl fur Numerische Mathematik Justus–Liebig–Universitaet Giessen, Germany

Abstract

Recently, the *shearlet transform* has gained a certain attention, being an extension to wavelets that is also capable of capturing *directional* singularities. While the continuous shearlet transform can be seen as a combination of shear operations and an anisotropic wavelet transform, thus allowing for an easy, intuitive and computationally efficient approach, a discrete counterpart is not so easily obtained by just discretizing the parameters. One reason for this difficulty is the lack of a natural extension of refinability.

Subdivision, on the other hand is a completely discrete, filter based approach to wavelets and automatically yields refinable functions as soon as the subdivision scheme converges. In the talk I will present a "directionally adaptive" generalization of subdivision schemes from which a refinement equation results that can be used to set up a directional multiresolution analysis for the construction of discrete shearlets.

A Tight Frame Approach for Missing Data Recovery in Images

Zuowei Shen

Department of Mathematics, National University of Singapore

Abstract

In many practical problems in image processing, the observed data sets are often incomplete in the sense that features of interest in the image are missing partially or corrupted by noise. The recovery of missing data from incomplete data is an essential part of any image processing procedures whether the final image is utilized for visual interpretation or for automatic analysis. In this talk, we will discuss our new iterative algorithm for image recovery for missing data which is based on tight framelet systems constructed by the unitary extension principle. We consider in particular few main applications in image processing, inpainting, impulse noise removal and super-resolution image reconstruction.

Symmetry of Shape Charts with Applications to Energy-optimizing Subdivision Algorithms

Georg Umlauf, Ingo Ginkel

Department of Computer Science, University of Kaiserslautern, Germany

Abstract

For subdivision surfaces, the so-called shape chart can be used to characterize the curvature behavior at an extraordinary point a priori from the initial control net. Of late, it has been used in different approaches to tune subdivision algorithms to handle the so-called hybrid shapes. For this the shape charts are computed numerically. In this talk, symmetries of shape charts are discussed that can be used to simplify the computations and to reduce the computation costs significantly. Based on these properties I will present a method to fair the limit surface of a subdivision algorithm locally around an extraordinary point. The goal of this method is to construct a sequence of spline rings around an extraordinary point by a modified subdivision matrix, such that each spline ring has minimal energy. The dominant six eigenvalues of the subdivision matrix have to satisfy linear and quadratic equality- and inequality-constraints in order to guarantee normal-continuity and bounded curvature at the extraordinary point. All smaller eigenvalues can be chosen arbitrarily within certain intervals and therefore can be used to optimize the shape of the subdivision surface by minimizing quadratic energy functionals. A suitable functional should contain third order measures and reasonably approximate the variation of curvature, since second order measures may lead to flat surfaces. Additionally, if the sub- and subsub-dominant eigenvalues vary within predefined intervals, C^1-regularity of the surface and locality of the stencils can be guaranteed, although eigenvectors are changed.

Ten Good Reasons for Using Splines for Signal/Image Processing

Michael Unser

EPFL Biomedical Imaging Group, Switzerland

Abstract

We argue that cardinal splines constitute an ideal framework for performing signal/image processing-the underlying philosophy being "thing analog, act digital". We show that multidimensional spline interpolation or approximation can be performed most efficiently using recursive digital filtering techniques. We highlight a number of "optimal" aspects of splines (in particular, polynomial ones) and discuss fundamental relations with: (1) Shannon's sampling theory, (2) linear system theory, (3) wavelet theory, (4) regularization theory, (5) estimation theory, and (6) stochastic processes (in particular, fractals). The practicality of the spline framework is illustrated with concrete image processing examples; these include derivative-based feature extraction, high-quality rotation and scaling, and (rigid body or elastic) image registration.

New Results in Nonlinear Subdivision

N. Dyn, P. Grohs, **J. Wallner**, A. Weinmann

Institute of Geometry, TU Graz. Kopernikusgasse 24, A 8010 Graz, Austria.

Abstract

For the analysis of convergence and smoothness of nonlinear subdivision schemes, the so-called method of proximity was poposed by [3]. It was subsequently employed to show further smoothness results in the univariate and multivariate case, for several different types of subdivision rules operating in manifolds and smooth groups. Recently this work has been extended to *approximation order*: [1] shows how C^k smoothness of a linear interpolatory rule can be used to show approximation order $k+1$ of analogous nonlinear subdivision rules. This results applies e.g. to nonlinear analogues of the Dubuc-Deslauriers schemes. Another important topic related to interpolatory schemes is the 'lazy' *wavelet coefficients* associated with them; here we could show that decay of wavelet coefficients corresponds to smoothness of data in both the univariate and multivariate cases [2]. A third topic of research is pursued by A. Weinmann in his Ph.D. thesis: he investigates convergence and smothness of nonlinar subdivision rules in the 2D non-regular case (with *extraordinary vertices* present), thus extending U. Reif's results [4]. In all cases, proximity inequalities play an important role.

References

[1] Nira Dyn, Philipp Grohs, and Johannes Wallner, *Approximation order of interpolatory nonlinear subdivision schemes*, preprint, TU Graz, November 2007.

[2] Philipp Grohs and Johannes Wallner, *Interpolatory wavelets for manifold-valued data*, preprint, TU Graz, March 2008.

[3] J. Wallner and N. Dyn, *Convergence and C^1 analysis of subdivision schemes on manifolds by proximity*, Comput. Aided Geom. Design **22** (2005), no. 7, 593–622.

[4] Andreas Weinmann, *Convergence and smoothness analysis of nonlinear stationary subdivision schemes in the presence of extaordinary points*, preprint, TU Graz, March 2008.

Smooth and Accurate Approximation of Manifold-Valued Data

Thomas Yu

Department of Mathematics, Drexel University
3141 Chestnut Street, Korman 269 Philadelphia, PA 19104

Abstract

There has been an emerging interest in developing an approximation theory for manifold-valued data, motivated by applications such as diffusion tensor imaging, motion capturing, etc.. In this talk, we address the following fundamental problem:
Let M be a smooth manifold equipped with a distance function d. For each smoothness factor $r > 0$ and approximation order $R > 0$, is there an approximation operator $A_{h;r,R}$ such that the operator maps samples of any $f : R \to M$ on a grid of size h to an approximant $f_h : R \to M$ with the properties that

a) $sup_x d(f_h(x), f(x)) = O(h^R)$ whenever f is a bounded C^R function, and
b) f_h is C^r smooth ?

We show that subdivision methods furnish a constructive method for solving this problem.

Polynomial Reproduction in Vector Subdivision: New Results on the Unstable Case

Georg Zimmermann

University of Hohenheim, Germany

Abstract

In subdivision, the degree of exactness, i.e., the maximal degree of polynomials that can be generated, plays an important role. For the stable case, there exist algorithms to determine both this quantity and simultaneously the vector data generating the polynomials. In the unstable case, the situation is more complicated since naturally, this data is not unique. We present new developments in this direction.

Speakers and participants

1. Carolina Beccari,
   University of Bologna, Italy, beccari@dm.unibo.it
2. Gabriella Bretti,
   University of Roma "La Sapienza", Italy, g.bretti@iac.cnr.it
3. **Tom Cashman** : May 1, 16:30-16:55,
   University of Cambridge, England, tc270@cam.ac.uk
4. **Maria Charina** : May 3 16:30-16:55,
   University of Dortmund, Germany, mcharina@Mathematik.Uni-Dortmund.DE
5. **Charles K. Chui** : May 2 9:00-9:50,
   University of Missouri and Stanford University, USA, ckchui@stanford.edu
6. Costanza Conti,
   University of Firenze, Italy, costanza.conti@unifi.it
7. **Mariantonia Cotronei** : May 3, 17:30-18:20,
   University Med. of Reggio Calabria, Italy, mariantonia.cotronei@unirc.it
8. Nira Dyn,
   Tel Aviv University, Israel, dyn@post.tau.ac.il
9. **Serge Dubuc** : May 4, 9:00-9:50,
   University of Montreal, Canada, dubucs@dms.umontreal.ca
10. **Tim Goodman** : May 1, 9:00-9:50,
    University of Dundee, Scotland, UK, tgoodman@maths.dundee.ac.uk
11. Laura Gori,
    University of Roma "La Sapienza", Italy, gori@dmmm.uniroma1.it
12. **Philipp Grohs** : May 2, 16:30-16:55,
    Graz University of Technology Austria, of Graz, Austria, grohs@geometrie.tuwien.ac.at
13. Nicola Guglielmi,
    University of L'Aquila, Italy, guglielm@univaq.it
14. **Bin Han** : May 3, 15:30-16:20,
    University of Alberta, Canada, bhan@gpu.srv.ualberta.ca
15. Olga Holtz,
    Berlin University of Technology, Germany, holtz@math.tu-berlin.de

16. **Kai Hormann** : May 4, 10:00-10:50,
Clausthal University of Technology, Germany, hormann@in.tu-clausthal.de

17. **Kurt Jetter** : May 3, 11:30-12:20,
University of Hohenheim, Germany, Prof.Kurt.Jetter@uni-hohenheim.de

18. **David Levin** : May 3, 9:00-9:50
Tel Aviv University, Israel, levin@post.tau.ac.il

19. Carla Manni,
University of Roma "Tor Vergata", Italy, manni@manni.uniroma2.it

20. **Jean Louis Merrien** : May 1, 17:30-18:20,
INSA de Rennes, France, Jean-Louis.Merrien@insa-rennes.fr

21. **Bernhard Mößner** : May 4, 12:00-12:25,
University of Freiburg, Germany, Mößner@mathematik.uni-freiburg.de

22. Enza Pellegrino,
University of L'Aquila, Italy, enza.pellegrino@ing.univaq.it

23. **Francesca Pitolli** : May 1, 18.30-18:55,
University of Roma "La Sapienza", Italy, pitolli@dmmm.uniroma1.it

24. Sara Remogna,
University of Torino, Italy, sara.remogna@unito.it

25. **Lucia Romani** : May 4, 11:30-11:55,
University of Milano Bicocca, Italy, lucia.romani@unimib.it

26. **Amos Ron** : May 1, 10:00-10:50,
University of Wisconsin - Madison, USA, amos@cs.wisc.edu

27. Milvia Rossini,
University of Milano Bicocca, Italy, milvia.rossini@unimib.it

28. **Malcolm Sabin** : May 1, 15:30-16:20,
Cambridge University, England, malcolm@geometry.demon.co.uk

29. Elisabetta Santi,
University of L'Aquila, Italy, esanti@ing.univaq.it

30. **Tomas Sauer** : May 2, 11:30-12:20,
University of Giessen, Germany, Tomas.Sauer@math.uni-giessen.de

31. **Zuowei Shen** : May 2, 10:00-10:50,
National University of Singapore, matzuows@nus.edu.sg

32. **Georg Umlauf** : May 3, 10:00-10:50,
University of Kaiserslautern, Germany, umlauf@informatik.uni-kl.de

33. **Michael Unser** : May 1, 11.30-12:20,
EPFL Biomedical Imaging Group, Switzerland, michael.unser@epfl.ch

34. **Johannes Wallner** : May 2, 15:30-16:20,
Graz University of Technology, Austria, j.wallner@tugraz.at

35. **Thomas Yu** : May 2, 17:30-18:20,
Drexel University, Philadelphia, PA, USA yut@drexel.edu

36. **Georg Zimmermann** : May 3, 18.30-18:55,
University of Hohenheim, Germany, Georg.Zimmermann@uni-hohenheim.de