**Compito 1**

E1.

Per il teorema dei residui il valore dell'integrale $\mathcal{I}$ è dato

$$\mathcal{I} = 2\pi i\left(\mathcal{R}es\left(\frac{e^z}{(z^2+9)(z^2+1)}, i\right) + \mathcal{R}es\left(\frac{e^z}{(z^2+9)(z^2+1)}, -i\right)\right) = \frac{\pi}{8}(e^i - e^{-i}) = \frac{\pi}{4} i \sin 1$$

E2.

$$s^2 Y(s) - 1 = \frac{Y(s)}{s^2}$$

$$\left(s^2 - \frac{1}{s^2}\right) Y(s) = 1$$

$$Y(s) = \frac{s^2}{s^4 - 1} = \frac{s^2}{(s-1)(s+1)(s-i)(s+i)}$$

$$s_1 = 1,\ s_2 = -1,\ s_3 = i,\ s_4 = -i$$

$$y(t) = \sum_{j=1}^{4} \mathcal{R}es\left(\frac{e^{st}s^2}{(s^4-1)}, s_j\right) =$$

$$\mathcal{R}es\left(\frac{e^{st}s^2}{(s^4-1)}, 1\right) + \mathcal{R}es\left(\frac{e^{st}s^2}{(s^4-1)}, -1\right) + \mathcal{R}es\left(\frac{e^{st}s^2}{(s^4-1)}, i\right) + \mathcal{R}es\left(\frac{e^{st}s^2}{(s^4-1)}, -i\right) =$$

$$\frac{1}{2}\sin t + \frac{1}{2}\sin ht$$

D1. (ii)

$$f(z) = \sum_{n=0}^{+\infty} (-1)^n \frac{z^{2n-2}}{(2n)!} + \sum_{n=0}^{+\infty} (-1)^n \frac{z^{n+1}}{5^{n+1}} - \sum_{n=0}^{+\infty} (-1)^n \frac{z^n}{5^{n+1}}$$

Parte singolare: $\frac{1}{z^2}$.

Polo doppio con $res(f(z), 0) = 0$

Parte regolare: $(-\frac{1}{2} - \frac{1}{5}) + (\frac{1}{5} + \frac{1}{25})z + (\frac{1}{4!} - \frac{1}{25} - \frac{1}{125})z^2 + (\frac{1}{125} + \frac{1}{625})z^3 + ....$

**Compito 2**

E1.

Per il teorema dei residui il valore dell'integrale $\mathcal{I}$ è dato

$$\mathcal{I} = 2\pi i\left(\mathcal{R}es\left(\frac{e^z}{(z^2-25)(z^2-1)}, 1\right) + \mathcal{R}es\left(\frac{e^z}{(z^2-25)(z^2-1)}, -1\right)\right) = \frac{\pi}{12}\sin 1$$

E2.

$$s^2 Y(s) - s = \frac{Y(s)}{s^2}$$

$$\left(s^2 - \frac{1}{s^2}\right) Y(s) = s$$

$$Y(s) = \frac{s^3}{s^4 - 1} = \frac{s^3}{(s-1)(s+1)(s-i)(s+i)}$$

$$s_1 = 1,\ s_2 = -1,\ s_3 = i,\ s_4 = -i$$

$$y(t) = \sum_{j=1}^{4} \mathcal{R}es\left(\frac{e^{st}s^3}{(s^4-1)}, s_j\right) =$$

$$\mathcal{R}es\bigg(\frac{e^{st}s^3}{(s^4-1)},1\bigg)+\mathcal{R}es\bigg(\frac{e^{st}s^3}{(s^4-1)},-1\bigg)+\mathcal{R}es\bigg(\frac{e^{st}s^3}{(s^4-1)},i\bigg)+\mathcal{R}es\bigg(\frac{e^{st}s^3}{(s^4-1)},-i\bigg)=$$
$$\frac{1}{2}\cosh t+\frac{1}{2}\cos t$$

D1. (ii)

$$f(z)=\frac{1}{z^3}-\sum_{n=0}^{+\infty}(-1)^n\frac{z^{2n-3}}{(2n)!}-\sum_{n=0}^{+\infty}\frac{z^{n+1}}{3^{n+1}}-\sum_{n=0}^{+\infty}\frac{z^n}{3^{n+1}}$$

Parte singolare: $\frac{1}{2z}$.
Polo semplice con $res(f(z),0)=\frac{1}{2}$
Parte regolare: $-\frac{1}{3}+(-\frac{1}{9}-\frac{1}{3}-\frac{1}{4!})z+(-\frac{1}{9}-\frac{1}{27})z^2+(\frac{1}{6!}-\frac{1}{27}-\frac{1}{81})z^3+....$

**L' esercizio E3 ( e la domanda D2 ) sono comuni ai due compiti.**

E3.
La funzione $f(x)$ è dispari, pertanto $a_0=0$, $a_k=0\,\forall k\in N$

$$b_k=\frac{1}{\pi}\bigg[\int_{-\pi}^{0}-x^2\sin kxdx+\int_0^{\pi}x^2\sin kxdx\bigg]=\frac{1}{\pi}\bigg[-\int_{\pi}^{0}-x^2\sin k(-x)dx+\int_0^{\pi}x^2\sin kxdx\bigg]=$$
$$\frac{1}{\pi}\bigg[\int_0^{\pi}x^2\sin kxdx+\int_0^{\pi}x^2\sin kxdx\bigg]=\frac{2}{\pi}\int_0^{\pi}x^2\sin kxdx$$

$$\int x^2\sin kxdx=-\int x^2\frac{1}{k}(\cos kx)'dx=-\frac{1}{k}x^2(\cos kx)+\frac{2}{k}\int x\cos kxdx$$

$$\int_0^{\pi}x^2\sin kxdx=-\frac{1}{k}x^2(\cos kx)|_0^{\pi}+\frac{2}{k}\int_0^{\pi}x\cos kxdx=-\frac{1}{k}\pi^2(-1)^k+\frac{2}{k^2}\int_0^{\pi}x(\sin kx)'dx=$$
$$=-\frac{1}{k}\pi^2(-1)^k+(\frac{2}{k^3}(-1)^k-\frac{2}{k^3})$$

Quindi indicata con $\mathcal{S}_1(x)$ la serie di Fourier relativa a $g$ e $\mathcal{S}_2(x)$ la serie di Fourier relativa a $f$, si ha

$$\mathcal{S}_1(x)=\frac{8}{\pi}\sum_{k=1}^{+\infty}\frac{1}{(2k-1)^3}\sin(2k-1)x$$
$$\mathcal{S}_2(x)=\frac{2}{\pi}\sum_{k=1}^{+\infty}\bigg(-\frac{1}{k}\pi^2(-1)^k+\frac{2}{k^3}(-1)^k-\frac{2}{k^3}\bigg)\sin kx$$
$$\mathcal{S}_1(\pi)=0$$
$$\mathcal{S}_2(\pi)=0$$