Esercizio 1 (Runge)

C. Simpson $h = 0.5$

$$\int_0^1 f(x)dx \simeq \frac{h}{3}\left[f(0) + 4f(0.5) + f(1)\right] =$$

$$= \frac{1}{6}\left[0 + 4 \cdot 0.1421 + 0.8417\right] = 0.2350167 =$$

$$= A_h$$

parabola $h = 0.25 = \frac{1}{4}$

$$\int_0^1 f(x)dx \simeq \frac{h}{3}\left[f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1)\right] =$$

$$= \frac{1}{12}\left[0 + 4 \cdot 0.0723 + 2 \cdot 0.1421 + 4 \cdot 0.2345 + 0.8417\right] = 0.196125 = A_{h/2}$$

$$R_{h/2} \simeq \frac{A_{h/2} - A_h}{15} = -0.0023502 = -0.002 \leq 0.5 \cdot 10^{-2}$$

$$W_{h/2} = 0.196125 - 0.0023502 = \underline{0.1937178}$$

1.2

Erreur totale = $R_{h/2} + E^*$

donc $|E^*| \leq 0.5 \cdot 10^{-4}$

$|E_{TOT}| \leq 0.2 \cdot 10^{-2} + 0.5 \cdot 10^{-4} = 0.245 \cdot 10^{-2}$

Exercício 2.

(2.1) $\begin{cases} y'(x) = y - 2x \\ y(0) = 1 \end{cases}$

$h = 0.1$, $x_0 = 0$, $x_1 = 0.1$

$y(0.1) \approx y_1 = y_0 + h f(x_0, y_0) = y_0 + h(y_0 - 2x_0) =$

$= 1 + 0.1(1 - 0) = 1.1$

$y(0.2) \approx y_2 = y_1 + h f(x_1, y_1) = y_1 + h(y_1 - 2x_1) =$

$= 1.1 + 0.1(1.1 - 0.2) =$

$= 1.1 + 0.1(0.9) = 1.19$

(2.2)

$$P(x) = L_0(x) y_0 + L_1(x) y_1 + L_2(x) y_2 =$$

$$\Rightarrow \quad P(0.15) = L_0(0.15) y_0 + L_1(0.15) y_1 + L_2(0.15) y_2$$

$$L_0(0.15) = \frac{(0.15 - 0.1)(0.15 - 0.2)}{(0.1)(-0.2)} = \frac{-0.05 \cdot 0.05}{0.02} = -0.125$$

$$L_1(0.15) = \frac{(0.15 - 0)(0.15 - 0.2)}{(0.1)(0.1 - 0.2)} = \frac{-0.15 \cdot 0.05}{-0.01} = +0.75$$

$$L_2(0.15) = \frac{(0.15-0)(0.15-0.1)}{(0.2)(0.2-0.1)} = 2$$

$$= \left(\frac{0.15 \cdot 0.05}{0.02}\right) = 0,375$$

$$\text{infine in } \begin{matrix} 1 & 1.1 & 1.19 \\ (0.0), & 0.1, & 0.2 \end{matrix}$$

$$P(0.15) = -0.125 \cdot 1 + 0.75 \cdot 1.1 + 0.375 \cdot 1.19 =$$

$$= 1.14625 \quad \text{(coerente con 1.1 e 1.19)}$$

$$|E(0.15)| = \frac{\pi_3(0.15)}{3!} |y'''(0.15)| \leq 10^{-3} \frac{|0.375|}{6} 3 = 0.1875 \cdot 10^{-3}$$

Poiché

$$\pi_3(0.15) = (0.15-0)(0.15-0.1)(0.15-0.2) =$$

$$= -0.15 \cdot 0.05 \cdot 0.05 = -0.375 \cdot 10^{-3}$$