- Calcolare $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^n}$.

...

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$ si ha

$$\left(1+\frac{1}{n}\right)^{n^n} = e^{n^n \log\left(1+\frac{1}{n}\right)} = e^{n^{n-1}\log\left(1+\frac{1}{n}\right)^n} \underset{n\to\infty}{\longrightarrow} +\infty .$$

- Calcolare il limite

$$\lim_{x\to 3^+} e^{\frac{\operatorname{sen}(x-3)}{(x-3)^2}} .$$

$$\lim_{x\to 3^+} e^{\frac{\operatorname{sen}(x-3)}{(x-3)^2}} = \lim_{x\to 3^+} e^{\frac{\operatorname{sen}(x-3)}{x-3}\,\frac{1}{x-3}} = +\infty .$$

3 - Dimostrare per induzione

$$\left(\log\left(\frac{17}{16}\pi\right)\right)^{n-1} \le n!, \qquad \text{per ogni } n\in\mathbf{N}, n\ge 1 .$$

per $n=1$ la disuguaglianza diventa $1=1$.

$$\left(\log\left(\frac{17}{16}\pi\right)\right)^{n} = \left(\log\left(\frac{17}{16}\pi\right)\right)^{n-1}\log\left(\frac{17}{16}\pi\right) \le n!\log\left(\frac{17}{16}\pi\right).$$

$$\left(\log\left(\frac{17}{16}\pi\right)\right)^{n} \le n!\log\left(\frac{17}{16}\pi\right) \le n!\,2 \le n!(n+1) = (n+1)!\,.$$

2 - Calcolare $\lim_{x\to 0}\frac{\operatorname{sen}(3x)}{5x+x^2}$.

$$\lim_{t\to 0}\frac{\operatorname{sen} t}{t} = 1 ,$$

$$\lim_{x\to 0}\frac{\operatorname{sen}(3x)}{5x+x^2} = \lim_{x\to 0}\frac{\operatorname{sen}(3x)}{3x}\,\frac{3}{5+x} = \frac{3}{5} .$$