

Discretaly

A workshop in Discrete Mathematics

Rome, February 1-2, 2018



Invited Speakers:

Krishnaswami Alladi (University of Florida)
Simeon Ball (Universitat Politècnica de Catalunya)
Andrea Burgess (University of New Brunswick)
Dieter Jungnickel (Universität Augsburg)
James Lepowsky (Rutgers University)
Mirko Primc (University of Zagreb)
Oriol Serra (Universitat Politècnica de Catalunya)
Leo Storme (Ghent University)
John Truss (University of Leeds)

Organizers:

Marco Buratti (Perugia), Stefano Capparelli (Roma Sapienza),
Francesca Merola (Roma Tre), Valentina Pepe (Roma Sapienza),
Tommaso Traetta (Perugia), Andrea Vietri (Roma Sapienza)

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SAPIENZA
UNIVERSITÀ DI ROMA

DIPARTIMENTO DI SCIENZE DI BASE
E APPLICATE PER L'INGEGNERIA

Discretaly Schedule

Thursday, February 1, 2018	Friday, February 2, 2018
9:30 - 10:00 Opening/Registration	9:30 - 10:10 O. Serra
10:00 - 10:40 D. Jungnickel	10:20 - 11:00 J. K. Truss
10:50 - 11:30 A. Burgess	11:10 - 11:40 Coffee break
11:45 - 12:00 G. Rinaldi	11:40 - 12:20 K. Alladi
12:00 - 12:15 A. Wassermann	12:30 - 12:45 S. Pagani
12:15 - 12:30 E. Brugnoli	12:45 - 14:00 Light Lunch – Buffet
12:30 - 12:45 Conference Photo	
12:45 - 14:00 Lunch – Buffet	
14:10 - 14:50 S. Ball	14:10 - 14:50 M. Primc
15:00 - 15:40 L. Storme	15:00 - 15:40 J. Lepowsky
15:50 - 16:15 Coffee Break	15:50 - 16:15 Coffee Break
16:15 - 16:30 S. Costa	16:15 - 16:30 A. Švob
16:30 - 16:45 F. Zullo	16:30 - 16:45 F. Pavese
16:45 - 17:00 M. Cavaleri	16:45 - 17:00 S. Mattheus
	17:00 - 17:15 D. A. Jaume

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Invited Talks

A partition theorem of Göllnitz and a new dual

Krishnaswami Alladi, University of Florida

One of the deepest results in the theory of partitions is a 1967 theorem of Göllnitz to the modulus 6. In 1995, Andrews, Gordon and I, obtained a generalization and three parameter refinement of Göllnitz' theorem by a new technique called the "method of weighted words" and found a remarkable q -hypergeometric key identity that is equivalent to the theorem. All this led to new connections between Göllnitz' theorem and several fundamental classical results in the theory of partitions and q -series. Related to Göllnitz' theorem are two hierarchies of partition theorems to the moduli $2^k - 1$, for $k > 1$ due to Andrews, the two hierarchies being duals of each other. Inspired by this, Andrews and I discovered a new dual to Göllnitz's theorem. After discussing the central role of Göllnitz' theorem in the theory of partitions, we will describe the construction of this dual from both a combinatorial and q -hypergeometric point of view.

On Sylvester problems for point sets in real space

Simeon Ball, Universitat Politècnica Catalunya

Let S be a set of n points in the real plane.

An *ordinary line* is a line incident with exactly two points of S . An *ordinary circle* is a circle incident with exactly three points of S . We are interested in proving structural results for sets S which span few ordinary lines or few ordinary circles. Problems of this type are called Sylvester problems and date back to the late 19th century.

In 2013, Green and Tao [2] classified all sets S for which the number of ordinary lines is less than Kn , where $K < c(\log \log n)^c$, for some constant c and sufficiently large n .

In [3], Lin et al prove that if S is not contained in a line or a circle then S spans at least $\frac{1}{4}n^2 - cn$ ordinary circles, for some constant c and sufficiently large n .

In this talk I shall present a proof of the following theorem from [1]. If S spans less than Kn^2 ordinary circles, for some $K = o(n^{\frac{1}{7}})$, then for n sufficiently large all but at most $O(K)$ points of S are contained in a curve of degree at most four. This theorem is deduced from the following 3-dimensional theorem inspired by the work of Green and Tao. Let S to be a set of n points in real 3-space. An *ordinary plane* is a plane incident with exactly three points of S . If any three points of S span a plane, S spans the whole space and spans less than Kn^2 ordinary planes, for some $K = o(n^{\frac{1}{7}})$ then, for n sufficiently large, all but at most $O(K)$ points of S are contained in the intersection of two quadrics.

References

- [1] S. Ball, On sets defining few ordinary planes, *Discrete and Computational Geometry*, to appear. [arXiv:1606.02138](https://arxiv.org/abs/1606.02138).
- [2] B. Green and T. Tao, On sets defining few ordinary lines, *Discrete and Computational Geometry*, 50 (2013) 409–468.
- [3] A. Lin, M. Makhul, H. N. Mojarad, J. Schicho, K. Swanepoel, F. de Zeeuw, On sets defining few ordinary circles, *Discrete and Computational Geometry*, to appear. [arxiv:1607.06597](https://arxiv.org/abs/1607.06597).

On 2-factorization problems, from Oberwolfach to Hamilton and Waterloo, and beyond

Andrea Burgess, University of New Brunswick

Suppose n mathematicians are attending a workshop at Oberwolfach, where the dining room has round tables of sizes m_1, m_2, \dots, m_k , with $m_1 + m_2 + \dots + m_k = n$. The Oberwolfach problem, first posed by Ringel, asks whether it is possible to make a seating arrangement over the successive days of the workshop so that each participant sits next to each other participant exactly once. In other words, given a 2-factor \mathcal{F} of K_n , does K_n admit an \mathcal{F} -factorization? If n is even, we instead factor $K_n - I$, the complete graph with the edges of a 1-factor removed.

The Oberwolfach problem has been extensively studied, and several variants have been introduced. In the Hamilton-Waterloo problem, the conference has two venues (Hamilton and Waterloo), so we seek a 2-factorization with α factors isomorphic to \mathcal{F}_1 and β factors isomorphic to \mathcal{F}_2 . Extending further, the generalized Oberwolfach problem considers t different 2-factors. In this talk, we discuss these and related 2-factorization problems and present some recent results.

This talk includes joint work with Peter Danziger and Tommaso Traetta.

On block codes of Steiner triple systems

Dieter Jungnickel, Universität Augsburg

In 1978, Doyen, Hubaut and Vandensavel proved that only the binary and ternary codes of Steiner triple systems can be interesting: for primes $p \neq 2, 3$, the p -ary code of any STS(v) has full rank v . They also proved that the 2-rank of a Steiner triple system on $2^n - 1$ points is at least $2^n - 1 - n$, with equality only for the classical point-line design in the projective geometry $PG(n-1, 2)$. Finally, they gave an analogous result for the ternary case, that is, for STS(3^n), where the classical examples are the point-line designs in ternary spaces. In 1995, Ed Assmus proved that the incidence matrices of all Steiner triple systems on v points which have the same 2-rank generate equivalent binary codes, and gave an explicit description of a generator matrix for such a code.

I will report on a systematic study of the binary and ternary block codes of Steiner triple systems in recent joint work with Vladimir Tonchev, where we obtained considerably simpler proofs for the results just described and also gave results analogous to those of Assmus for the ternary case. In all these cases, we provide explicit parity check matrices for the codes in question.

We have also applied these results to the enumeration problem for STS on $2^n - 1$ or 3^n points with a prescribed 2-rank or 3-rank, respectively. In particular, we could prove a general formula for the number of *distinct* STS($2^n - 1$) with 2-rank at most $2^n - 1 - n + t$ contained in the relevant code, which then leads to both lower and upper bounds for the number of *isomorphism classes* of STS($2^n - 1$) with 2-rank exactly (or at most) $2^n - 1 - n + t$. We also proved corresponding results for the ternary case. In both cases, the lower bounds appear to be quite strong and show the expected combinatorial explosion even for STS with small rank.

Our results provide the first two infinite families of 2-designs for which one has non-trivial lower and upper bounds for the number of non-isomorphic examples with a prescribed p -rank in almost the entire range of possible ranks. (The only cases where our bounds do not apply are for designs having full 2-rank v , or 3-rank $v - 1$.)

How can Capparelli's identities be "understood"?

James Lepowsky, Rutgers University, USA

Capparelli's identities, conjectured in his 1988 Rutgers Ph.D. thesis and subsequently proved in different ways, including by Capparelli, were the first new Rogers-Ramanujan-type identities predicted by application of the vertex-operator-theoretic twisted Z -algebra theory developed by Lepowsky-Wilson. They were in fact the first such identities that were not in the natural infinite family of Gordon-Andrews-Bressoud identities. They have also been placed into other vertex-operator-algebraic settings by Meurman-Primc and others. It is expected that a series of developments termed "motivated proofs" of generalized Rogers-Ramanujan identities should lead to new structure in the theory of intertwining operator algebras for generalized vertex operator algebras. I'll sketch why trying to "explain" Capparelli's identities in this way is an important problem.

Some new combinatorial identities related to representations of affine Lie algebras

Mirko Primc, University of Zagreb

In my talk, based on joint work with T. Šikić, I will describe some new combinatorial identities that appear for level 1 representations of symplectic affine Lie algebras, and some that (conjecturally) should appear for higher level representations.

Linear analogues of additive theorems

Oriol Serra, Universitat Politècnica de Catalunya

Inequalities relating the cardinality of a sumset with the cardinality of summands play a central role in additive combinatorics. Hou, Leng and Xian gave a version of one of the classical results in the area, the theorem of Kneser, in separable extensions of fields, where dimensions of subspaces play the role of cardinalities. One of the nice features of this dimension version of Kneser's theorem is that it gives the classical one as a corollary. The talk will discuss several results of the same nature where classical results in additive combinatorics are translated to the linear setting. In particular we will focus on a new combinatorial proof of a strengthening of the dimensional Kneser theorem and on the simpler example of an inverse theorem, the theorem of Vosper. The latter is related to an intriguing problem on Sidon spaces which has been tackled with the Delsarte linear programming method introduced to study maximum distance separating codes in the space of bilinear forms.

This is joint work with Christine Bachoc and Gilles.

Cameron-Liebler sets in geometrical settings

Leo Storme, Ghent University

Cameron-Liebler sets are one of the types of substructures in finite projective spaces, presently receiving a lot of attention. One of the nice aspects of Cameron-Liebler sets is that they can be defined in many equivalent ways; thus showing their intrinsic geometric relevance.

After a large number of results regarding Cameron-Liebler sets of lines in the 3-dimensional projective space $\text{PG}(3, q)$ [2], Cameron-Liebler sets of k -spaces in the $(2k + 1)$ -dimensional projective space $\text{PG}(2k + 1, q)$ were defined [4]. Here, links to Erdős-Ko-Rado results were used to obtain results on these Cameron-Liebler sets of k -spaces in $\text{PG}(2k + 1, q)$ [3].

These links motivated to initiate research aimed at defining Cameron-Liebler sets in finite classical polar spaces [1]. When defining Cameron-Liebler sets in a new setting, the first aim is to find a substructure which can be defined in similar equivalent ways as in the projective space setting. The second aim is to find examples and characterization results on these Cameron-Liebler sets in those new settings.

This talk will discuss Cameron-Liebler sets in finite projective spaces and in finite classical polar spaces. We present new equivalent definitions of these Cameron-Liebler sets. We present existence and non-existence results on Cameron-Liebler sets. One of the new aims is to use the newly discovered equivalent definitions of Cameron-Liebler sets to improve known results on Cameron-Liebler sets.

References

- [1] M. De Boeck, M. Rodgers, L. Storme, and A. Švob, Cameron-Liebler sets of generators in finite classical polar spaces, *J. Combin. Theory, Ser. A*, submitted.
- [2] A.L. Gavriluyk and K. Metsch, A modular equality for Cameron-Liebler line classes, *J. Combin. Theory, Ser. A*, 127 (2014) 224–242.
- [3] Chris Godsil and Karen Meagher, *Erdős-Ko-Rado Theorems: Algebraic Approaches*. Cambridge University Press 2015.
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Ehrenfeucht-Fraïssé games on linear orders

John K. Truss, University of Leeds

In an n -move Ehrenfeucht-Fraïssé game on relational structures A and B , players I and II play alternately. On each move player I chooses an element of one of the structures, and player II tries to ‘match it’ by choosing an element of the other structure. Player I does not have to choose from the same structure on each move. After n moves, points a_1, a_2, \dots, a_n have been chosen in A , and points b_1, b_2, \dots, b_n have been chosen in B , and player II *wins* if the map taking a_i to b_i for each i is an isomorphism of induced substructures. Under these circumstances, we say that A and B are n -equivalent, written $A \equiv_n B$. The main facts about this situation are as follows. A and B are n -equivalent if and only if they satisfy the same sentences of quantifier depth at most n . Hence \equiv_n is an equivalence relation, and A and B are elementarily equivalent if and only if they are n -equivalent for all n . If the language is finite, then for each n there are just finitely many \equiv_n -classes. We consider linear orders, and also linear orders with colours, concentrating on estimates for the lengths of optimal representatives for finite coloured orders, optimal representatives for ordinals and certain special scattered linear orders.

This is joint work with Feresiano Mwesigye.

Contributed Talks

The golden cow construction

Emanuele Brugnoli, University of Palermo

A $2 - (v, k, 1)$ *design* or *Steiner 2-design* is a pair (V, \mathcal{B}) where V is a set of v *points* and \mathcal{B} is a collection of k -subsets (*blocks*) of V such that any 2-subset of V is contained in exactly one block. Such a design is said to be *cyclic* if $V = \mathbb{Z}_v$ and \mathcal{B} is invariant under addition modulo v .

To establish the number $\text{NC}(v, k)$ of cyclic $2 - (v, k, 1)$ designs is in general not feasible and very little is known about this number. By “playing” with $(v, k, 1)$ *difference families*, some lower bounds on $\text{NC}(v, k)$ are given. In particular, for primes $p = 6n + 1$ with $p \equiv \pm 1 \pmod{5}$, a construction involving the *golden ratio* of \mathbb{Z}_p and the *Narayana cows sequence* is shown to give $\text{NC}(p, 3) > 2^{3n/2}$.

This is joint work with Marco Buratti and Mikhail E. Muzychuk.

Wreath product of graphs: spectrum and topological indices

Matteo Cavaleri, Università degli Studi Niccolò Cusano

In this talk we give a brief introduction to the wreath product of graphs, which is a graph analogue of the group wreath product. This construction is intensively studied in Probability and Geometric group theory; however, our approach is mostly combinatorial. Using the description of the adjacency matrix given by D'Angeli and Donno, we give an explicit computation of the spectrum in some special cases. Moreover, we are able to characterize some topological indices for graph wreath products, which are of great interest for their applications in Mathematical chemistry, as the Zagreb indices, the Wiener index and the Szeged index: we provide explicit formulas in term of the corresponding indices of the factor graphs. Finally, we describe the Antipodal graph of wreath products of graphs.

The talk is based on joint work with F. Belardo and A. Donno.

New 2-designs via strong difference families

Simone Costa, University of Brescia

A $2-(v, k, \lambda)$ design is a pair (V, \mathcal{B}) where V is a set of v points and \mathcal{B} is a collection of k -subsets of V (blocks) such that every 2-subset of V is contained in exactly λ blocks. Such a design is *resolvable* if there exists a partition of \mathcal{B} into classes (*parallel classes*) each of which is in its turn a partition of V . Despite the fact that the literature on 2-designs and resolvable 2-designs is huge, for their existence we still have many open cases even for “small” values of k . In this talk I will show how we have been able in [2, 3] to solve fifteen of these open cases - in each of which $k = 7, 8$ or 9 - by using the method of *strong difference families* introduced by Buratti [1]. The same method enabled us to establish the existence of $2-(v, 13, 1)$ and $2-(v, 17, 1)$ designs for infinitely many new values of v .

This is joint work with Tao Feng and Xiaomiao Wang.

References

- [1] M. Buratti, Old and new designs via difference multisets and strong difference families, *J. Combin. Des.*, 7 (1999) 406–425.
- [2] S. Costa, T. Feng, and X. Wang, New 2-designs from strong difference families, in press on *Finite Fields Appl.*
- [3] S. Costa, T. Feng, and X. Wang, Frame difference families and resolvable balanced incomplete block designs, submitted.

Combinatorial interpretation of the Drazin inverse of trees.

Daniel A. Jaume, Universidad Nacional de San Luis

Let T be a tree, we denote its adjacency matrix by A . Since A is symmetric, if A is singular its Drazin inverse is the unique matrix D satisfying the relations: $AD = DA$, $ADA = A$, and $DAD = D$.

The set of all maximum matchings in T is denoted by $\mathcal{M}(T)$, and $m(G) := |\mathcal{M}(G)|$. Let P be a path in T , and M a matching in T . Then P is an **alternating** path with respect to M if it has edges that are alternately free and matched. An alternating path P with respect to a matching M is said a **coaugmenting** path with respect to the matching M if P starts and ends at matched vertices (by edges of $M \cap E(P)$). The set of all coaugmenting paths in G with respect to M is denoted by $CoAugGM$. Let T be a tree, and $i, j \in V(T)$. We write iPj for the unique path between i and j in T . By $\mathcal{M}(T, i, j)$ we mean the set of all the maximum matchings M such that iPj is a coaugmenting path with respect to M .

$$\mathcal{M}(T, i, j) := \{M \in \mathcal{M}(T) : iPj \in CoAug(G, M)\}$$

Definition 1. Let T be a tree of order n . The **combinatorial Drazin inverse** of T is the n by n matrix $R(T) = (r_{ij})_{1 \leq i, j \leq n}$ defined by

$$r_{ij} := (-1)^{\lfloor \frac{d(i,j)}{2} \rfloor} \frac{m(T, i, j)}{m(T)}$$

The following is our main result:

Theorem 2. *Let T be a tree. Then $R(T)$ is the Drazin inverse of T .*

This generalizes the work of Pavlíková and Krč-Jediný, whose gave a combinatorial description of the inverse of nonsingular trees.

This is joint work with Rodrigo Sota.

The forbidden configuration problem for polar triangles

Sam Mattheus, Vrije Universiteit Brussel

Consider a finite projective plane and a polarity of this plane. A polar triangle is a set of three non-collinear points, such that the image of any of the points under the polarity contains the other two points. A natural extremal problem is to find the largest set of points avoiding this configuration. The focus will be on the case when the polarity is unitary, i.e. the absolute points are the points of a unital. First we will show an upper bound for any finite projective plane by methods from spectral graph theory, and then a lower bound when the plane is Desarguesian of even order, which asymptotically matches the upper bound. Finally, we will shortly discuss how these results transfer to the case of the Figueroa plane.

This is joint work with Francesco Pavese.

Regions of uniqueness in discrete tomography

Silvia M.C. Pagani, Politecnico di Milano

Discrete tomography aims to reconstruct the internal of an object by means of the knowledge of its projections, taken along given directions. The tomographic problem is ill-posed in general, so one of the main lines of research consists in finding the conditions that can ensure uniqueness of reconstruction.

We address the problem from a local perspective, and seek which parts of the object can be uniquely determined by the given set of directions. Such parts are called regions of uniqueness.

In this talk we present some results concerning the combinatorial characterization of the regions of uniqueness for different sets of directions. Furthermore, we suggest how to choose the set of directions, such that its cardinality is close to minimality.

Strongly regular graphs from classical generalized quadrangles

Francesco Pavese, Politecnico di Bari

A *strongly regular graph* $srg(v, k, \lambda, \mu)$ is a graph with v vertices such that each vertex lies on k edges, any two adjacent vertices have exactly λ common neighbours, and any two non-adjacent vertices have exactly μ common neighbours.

A *generalized quadrangle of order* (s, t) ($GQ(s, t)$ for short) is an incidence structure $(\mathcal{P}, \mathcal{B}, \mathcal{I})$ of points and lines with the properties that any two points (lines) are incident with at most one line (point), every point is incident with $t + 1$ lines, every line is incident with $s + 1$ points, and for any point P and line l which are not incident, there is a unique point on l collinear with P .

The *point-graph* of a $GQ(s, t)$ has vertices corresponding to the elements of \mathcal{P} . Two vertices are adjacent if the corresponding points of \mathcal{P} lie on a line of \mathcal{B} . It is well known that such a graph is a strongly regular graph with parameters $v = (s + 1)(st + 1)$, $k = st + s$, $\lambda = s - 1$, $\mu = t + 1$ and is said to be *geometric*. However, there may be graphs with such a parameter set that do not arise in this way. A strongly regular graph with the above parameters (for some s and t) is called *pseudo-geometric*.

If s is an even power of a prime and t equals s or $s\sqrt{s}$, by modifying in a suitable way the point-graph of a classical $GQ(s, t)$, it is shown the existence of pseudo-geometric strongly regular graphs having the same parameters of the point-graph of a $GQ(s, t)$, that are not geometric.

Spanning tree decompositions of complete graphs orthogonal to 1-factorizations

Gloria Rinaldi, University of Modena and Reggio Emilia

A 1-factorization of a complete graph is a decomposition of the edge set into perfect matchings. Such a decomposition obviously exists only when the number of vertices is even, say $2n$. A spanning tree of K_{2n} is said to be orthogonal to a 1-factorization if it shares exactly one edge with each 1-factor. In 1996 R.A. Brualdi and S. Hollingsworth conjecture that, when $n \geq 3$, for each 1-factorization \mathcal{F} the edge set of K_{2n} can be partitioned into n edge disjoint spanning trees orthogonal to \mathcal{F} . We examine the state of art on this conjecture and show that in some cases the partition is possible.

Strongly regular graphs and groups

Andrea Švob, University of Rijeka

In this talk we will describe a construction of regular graphs admitting a transitive action of the group G . We apply this method to construct transitive strongly regular graphs from some classical finite simple groups (orthogonal, unitary and linear groups). The details about the obtained strongly regular graphs will be given.

This is joint work with Dean Crnković.

q -analogs of group divisible designs

Alfred Wassermann, University of Bayreuth

Group divisible designs are well-studied combinatorial objects. In this talk, we introduce q -analogs of group divisible designs (q -GDDs).

Definition 3. Let K and G be sets of positive integers and let λ be a positive integer. The q -analog of a group divisible design of index λ and order v is a triple $(\mathcal{V}, \mathcal{G}, \mathcal{B})$, where \mathcal{V} is a vector space over $\text{GF}(q)$ of dimension v , \mathcal{G} is a vector space partition of \mathcal{V} into subspaces (groups) whose dimensions lie in G , and \mathcal{B} is a family of subspaces (blocks) of \mathcal{V} that satisfy

1. if $B \in \mathcal{B}$ then $\dim B \in K$,
2. every 2-dimensional subspace of \mathcal{V} occurs in exactly λ blocks or one group, but not both, and
3. $\#\mathcal{G} > 1$.

A q -GDD is g -uniform, if all groups have the same dimension g .

We give necessary conditions on the parameters for the existence of q -GDDs. Interestingly enough, one of these restrictions is connected to the existence of q^r -divisible linear codes. We also present a list of uniform q -GDDs for $K = \{k\}$ which we constructed with the Kramer-Mesner method.

This is joint work with Marco Buratti, Sascha Kurz and Anamari Nakić.

MRD-codes and a new criteria for generalized twisted Gabidulin codes

Ferdinando Zullo, Università degli Studi della Campania “Luigi Vanvitelli”

The set of $m \times n$ matrices $\mathbb{F}_q^{m \times n}$ over \mathbb{F}_q is a metric \mathbb{F}_q -space with rank metric distance defined by $d(A, B) = \text{rk}(A - B)$ for $A, B \in \mathbb{F}_q^{m \times n}$. A subset $\mathcal{C} \subseteq \mathbb{F}_q^{m \times n}$ is called a **rank distance code**. The minimum distance of \mathcal{C} is

$$d(\mathcal{C}) = \min\{d(A, B) : A, B \in \mathcal{C}, A \neq B\}.$$

In [2] the Singleton bound for an $m \times n$ rank metric code \mathcal{C} with minimum rank distance d was proved:

$$\#\mathcal{C} \leq q^{\max\{m,n\}(\min\{m,n\}-d+1)}.$$

If this bound is achieved, then \mathcal{C} is an **MRD-code**.

When \mathcal{C} is an \mathbb{F}_q -linear subspace of $\mathbb{F}_q^{m \times n}$, we say that \mathcal{C} is an **\mathbb{F}_q -linear code** and the dimension $\dim_q(\mathcal{C})$ is defined to be the dimension of \mathcal{C} as a subspace over \mathbb{F}_q . If d and k are, respectively, the minimum distance and the dimension of \mathcal{C} , we say that \mathcal{C} has parameters $[m \times n, k, d]_{\mathbb{F}_q}$.

In [1], a new example of MRD-code with parameters $[6 \times 6, 2, 5]_{\mathbb{F}_q}$ with $q \equiv 0, \pm 1$ is presented, while in [3] is derived a criteria to establish whether an MRD-code with maximum left idealiser is equivalent to a generalized twisted Gabidulin code.

References

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- [2] P. Delsarte, Bilinear forms over a finite field, with applications to coding theory, *J. Combin. Theory Ser. A*, 25 (1978) 226–241.
- [3] L. Giuzzi and F. Zullo, New criteria for generalized twisted Gabidulin codes, manuscript.

List of Participants

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