

1. Trovare le ultime due cifre di  $513^{323}$ .

**Soluzione.** Lavoriamo modulo 100.

$$\begin{aligned} 513^{323} &\equiv 13^{323} \\ &13^{256} 13^{64} 13^3 \end{aligned}$$

Ora abbiamo

$$13^2 \equiv 69, 13^4 \equiv 69^2 \equiv 61, 13^8 \equiv 61^2 \equiv 21$$

$$13^{16} \equiv 21^2 \equiv 41, 13^{32} \equiv 41^2 \equiv 81, 13^{64} \equiv 81^2 \equiv 61$$

$$13^{128} \equiv 21, 13^{256} \equiv 41$$

Infine  $13^{64} \equiv 61$  e quindi  $13^{256} 13^{64} 13^3 \equiv 41 \cdot 61 \cdot 97 \equiv 97$ .